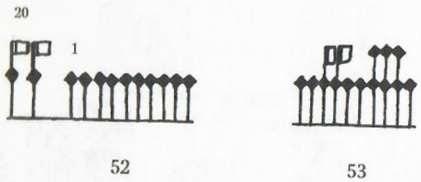

FIG. 22.19. *Numerical representations from a Zapotec painting made by order of the Spanish colonial authorities in 1540. It shows graphical conventions common to Zapotec, Mixtec, and Aztec numeral systems.*

So it seems certain that "ordinary" Maya numerals must also have been strictly vigesimal and based on the additive principle. It can be safely assumed that a circle or dot was used to represent the unity (the sign is common to all Central American cultures, and derives from the use of the cocoa bean as the unit of currency), that there was a special sign, maybe similar to the "hatchet" used by other Central American cultures, for the base (20), and other specific signs for the square of the base (400) and the cube (8,000), etc.

As we shall see below, it is also quite probable that, like the Zapotecs, the Maya introduced an additional sign for 5, in the form of a horizontal line or bar.

Even though no trace of it remains, we can reasonably assume that the Maya had a numeral system of this kind, and that intermediate numbers were figured by repeating the signs as many times as was needed. But that kind of numeral system, even if it works perfectly well as a recording device, is of no use at all for arithmetical operations. So we must assume that the Maya and other Central American civilisations had an instrument similar to the abacus for carrying out their calculations.

The Inca of South America certainly did have a real abacus, as shown in Fig. 22.20. The Spaniards were amazed at the speed with which Inca accountants could resolve complex calculations by shifting ears of maize, beans or pebbles around twenty "cups" (in five rows of four) in a tray or table, which could be made of stone, earthenware or wood, or even just laid out in the ground. Inca civilisation was obviously quite different from the Maya world, but it did have one thing in common: a method of recording numbers and tallies (the *quipus*, or knotted string) that was entirely unsuitable for performing arithmetical operations. For that reason the Inca were obliged to devise a different kind of operating tool.

FIG. 22.20. *Document proving the use of the abacus amongst the Peruvian and Ecuadorian Incas. It shows a* quipucamayoc *manipulating a quipu and on his right a counting table. From the Peruvian Codex of Guaman Poma de Ayala (16th century), Royal Library, Copenhagen*

THE PLACE-VALUE SYSTEM OF "LEARNED" MAYA NUMERALS

The only numerical expressions of the Maya that have survived are in fact not of the ordinary or practical kind, but astronomical and calendrical calculations. They are to be found in the very few Maya manuscripts that exist, and most notably in the Dresden Codex, an astronomical treatise copied in the eleventh century CE from an original that must have been three or four centuries older.

What is quite remarkable is that Maya priests and astronomers used a numeral system with base 20 which possessed a true zero and gave a specific value to numerical signs according to their position in the written expression. The nineteen first-order units of this vigesimal system were represented by very simple signs made of dots and lines: one, two, three and four dots for the numbers 1 to 4; a line for 5, one, two, three and four dots next to the line for 6 to 9; two lines for 10, and so on up to 19:

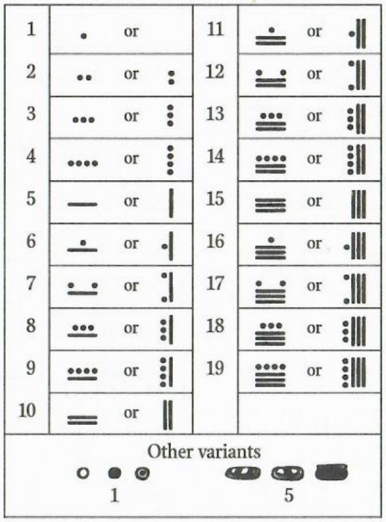

FIG. 22.21. *The first nineteen units in the numeral system of the Maya priests*

Numbers above 20 were laid out vertically, with as many "floors" as there were orders of magnitude in the number represented. So for a number involving two orders, the first order-units were expressed on the first or "bottom floor" of the column, and the second-order units on the "second floor". The numbers 21 (= 1 × 20 + 1) and 79 (3 × 20 + 19) were written thus:

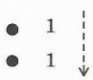

FIG. 22.22.

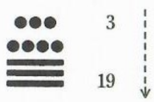

FIG. 22.23.

The "third floor" should have been used for values twenty times as great as the "second floor" in a regular vigesimal system. Just as in our decimal system the third rank (from the right) is reserved for the hundreds (10 × 10 = 100), so in Maya numbering the third level should have counted the "four hundreds" (20 × 20 = 400). However, in a curious irregularity that we will explain below, the third floor of Mayan astronomical numerals actually represented multiples of 360, not 400. The following expression:

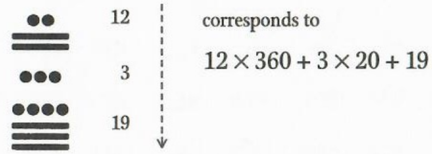

FIG. 22.24.

actually meant 12 × 360 + 3 × 20 + 19 = 4,399, and not 12 × 400 + 3 × 20 + 19 = 4,879!

Despite this, higher floors in the column of numbers were strictly vigesimal, that is to say represented numbers twenty times as great as the immediately preceding floor. Because of the irregularity of the third position, the fourth position gave multiples of 7,200 (360 × 20) and the fifth gave multiples of 144,000 (20 × 7,200) – and not of 8,000 and 160,000.

A four-place expression can thus be resolved by means of three multiplications and one addition, thus:

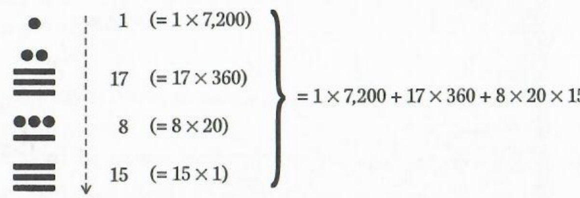

FIG. 22.25.

So that each numeral would be in its right place even when there were no units to insert in one or another of the "floors", Mayan astronomers invented a zero, a concept which they represented (for reasons we cannot pierce) by a sign resembling a snail-shell or sea-shell.

For instance, a number which we write as 1,087,200 in our decimal place-value system and which corresponds in Mayan orders of magnitude to 7 × 144,000 + 11 × 7,200 and no units of any of the lower orders of 360, 20 or 1, would be written in Maya notation thus:

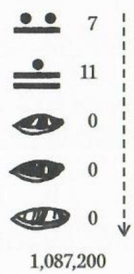

FIG. 22.26.

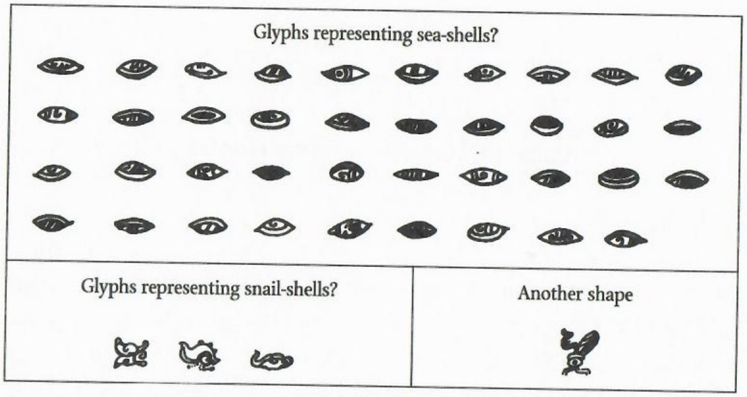

FIG. 22.27.

We can see the system in operation in these very interesting numerical expressions in the Dresden Codex:

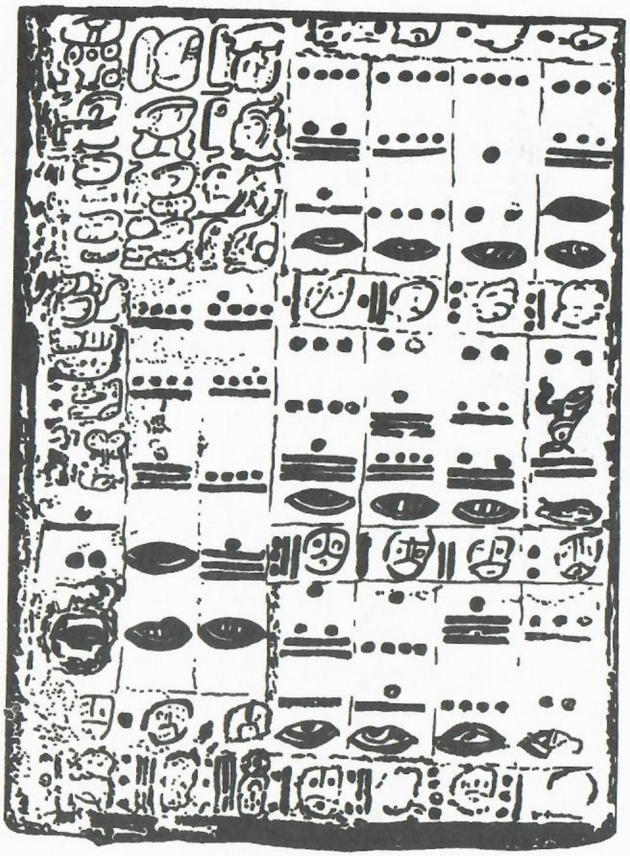

FIG. 22.28. *The Dresden Codex, p. 24 (part). Sächsische Landesbibliothek, Dresden*

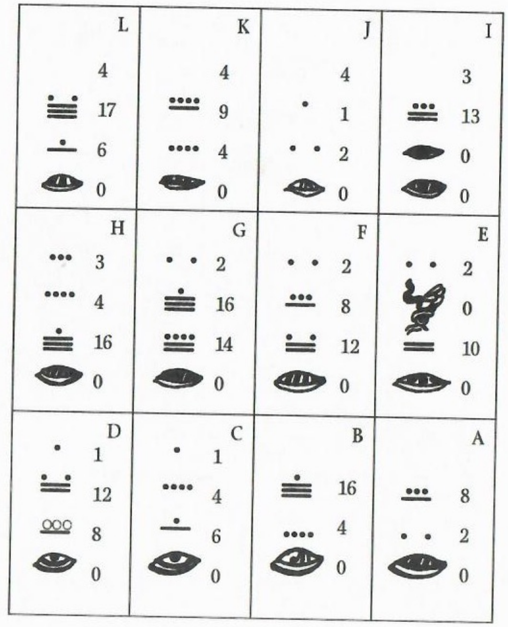

FIG. 22.29. *Transcriptions of the numerals on the right-hand side of Fig. 22.28*

Each of these expressions in Mayan astronomical notation refers to a number of days (we know this from the context) and gives the following set of equivalences:

A =	[8; 2; 0] =	2,920 =	1 × 2,920 =	5 × 584
B =	[16; 4; 0] =	5,840 =	2 × 2,920 =	10 × 584
C =	[1; 4; 6; 0] =	8,760 =	3 × 2,920 =	15 × 584
D =	[1; 12; 8; 0] =	11,680 =	4 × 2,920 =	20 × 584
E =	[2; 0; 10; 0] =	14,600 =	5 × 2,920 =	25 × 584
F =	[2; 8; 12; 0] =	17,520 =	6 × 2,920 =	30 × 584
G =	[2; 16; 14; 0] =	20,440 =	7 × 2,920 =	35 × 584
H =	[3; 4; 16; 0] =	23,360 =	8 × 2,920 =	40 × 584
I =	[3; 13; 0; 0] =	26,280 =	9 × 2,920 =	45 × 584
J =	[4; 1; 2; 0] =	29,200 =	10 × 2,920 =	50 × 584
K =	[4; 9; 4; 0] =	32,120 =	11 × 2,920 =	55 × 584
L =	[4; 17; 6; 0] =	35,040 =	12 × 2,920 =	60 × 584

So this series is nothing other than a table of the synodic revolutions of Venus, calculated by Mayan astronomers as 584 days.

This gives us two indisputable proofs of the mathematical genius of Maya civilisation:

- it shows that they really did invent a place-value system;
- it shows that they really did invent zero.

These are two fundamental disoveries that most civilisations failed to make, including especially Western European civilisation, which had to wait until the Middle Ages for these ideas to reach it from the Arabic world, which had itself acquired them from India.

One problem remains: why was this system not strictly vigesimal, like the Mayas' oral numbering? For instead of using the successive powers of 20 (1, 20, 400, 8,000, etc.), it used orders of magnitude of 1, 20, $18 \times 20 = 360$, $18 \times 20 \times 20 = 7{,}200$, etc. In short, why was the third "floor" of the system occupied by the irregular number 360?

If Maya numerals had been strictly vigesimal, then its zero would have acquired operational power: that is to say, adding a zero at the end of a numerical string would have multiplied its value by the base. That is how it works in our system, where the zero is a true operational sign. For instance, the number 460 represents the product of 46 multiplied by the base. For the Maya, however, [1; 0; 0] is not the product of [1; 0] multiplied by the base, as the first floor gives units, the second floor gives twenties, but the third floor gives 360s. [1; 0] means precisely 20; but [1; 0; 0] is not 400 ($20 \times 20 + 0 +$), but 360. The number 400 had to be written as [1; 2; 0] or ($1 \times 360 + 2 \times 20 + 0$):

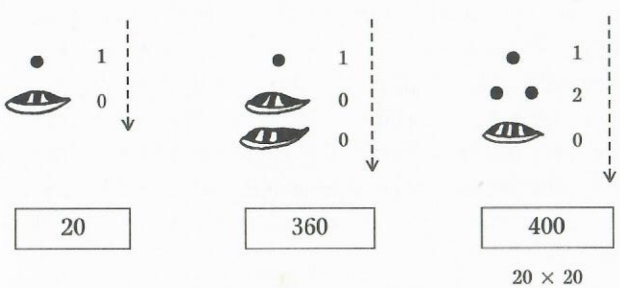

FIG. 22.30.

This anomaly deprived the Maya zero of any operational value, and prevented Mayan astronomers from exploiting their discovery to the full. We must therefore not confuse the Maya zero with our own, for it does not fulfil the same role at all.

A SCIENCE OF THE HIGH TEMPLES

To understand the odd anomaly of the third position in the Maya place-value system we have to delve deep into the very sources of Maya mathematics, and make a long but fascinating detour into Maya mysticism and its reckoning of time.

Maya learned numerals were not invented to deal with the practicalities of everyday reckoning – the business of traders and mere mortals – but to meet the needs of astronomical observation and the reckoning of time. These numerals were the exclusive property of priests, for Maya civilisation made the passing of time the central matter of the gods.

Maya science was practised in the high temples: astronomy was what the priests did. Mayan achievements in astronomy, including the invention of one of the best calendars the world has ever seen, were part and parcel of their mystical and religious beliefs.

The Maya did not think of time as a purely abstract means of ordering events into a methodical sequence. Rather, they viewed it as a supernatural phenomenon laden with all-powerful forces of creation and destruction, directly influenced by gods with alternately kindly and wicked intentions. These gods were associated with specific numbers, and took on shapes which allowed them to be represented as hieroglyphs. Each division of the Maya calendar (days, months, years, or longer periods) was thought of as a "burden" borne on the back of one or another of the divine guardians of time. At the end of each cycle, the "burden" of the next period of time was taken over by the god associated with the next number. If the coming cycle fell to a wicked god, then things would get worse until such time as a kindly god was due to take over. These curious beliefs supported the popular conviction that survival was impossible without learned mediators who could interpret the intentions of the irascible gods of time. The astronomer-priests alone could recognise the attributes of the gods, plot their paths across time and space, and thus determine times that would be controlled by kindly gods, or (as was more frequent) times when the number of kindly gods would exceed that of evil gods. It was an obsession for calculating periods of luck and good fortune over long time-scales, in the hope that such foreknowledge would enable people to turn circumstances to their advantage. [See C. Gallenkamp (1979)]