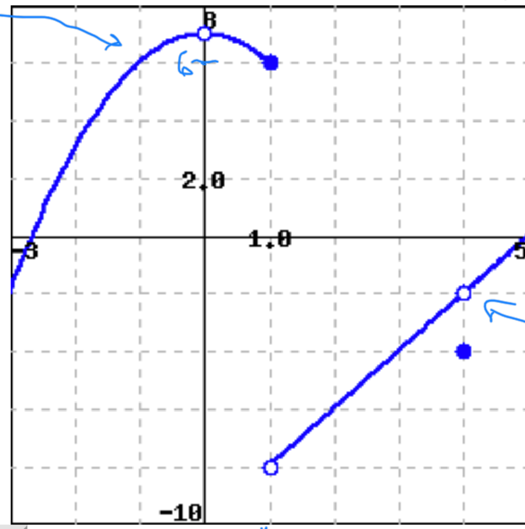
1. Limits on a piecewise graph.

Use the figure below, which gives a graph of the function $f(x)$, to give values for the indicated limits. If a limit does not exist, enter ***none***.

nice & continuous here

Out[500]=



hole doesn't affect points in the neighborhood

(a) $\lim\limits_{x\to -1} f(x) =$ 6 ⓘ help (limits)

(b) $\lim\limits_{x\to 0} f(x) =$ 7 ⓘ

(c) $\lim\limits_{x\to 1} f(x) =$ *none* ⓘ

Big jump - left & right don't agree

(d) $\lim\limits_{x\to 4} f(x) =$ -2 ⓘ

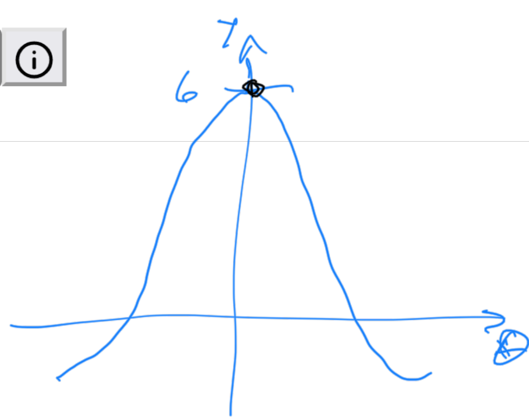
2. Estimating a limit graphically.

Use a graph to estimate the limit

$$\lim_{\theta\to 0} \frac{\sin(6\theta)}{\theta}.$$

Note: θ is measured in radians. All angles will be in radians in this class unless otherwise specified.

$\lim\limits_{\theta\to 0} \frac{\sin(6\theta)}{\theta} =$ 6 ⓘ



3. Limits for a piecewise formula.

For the function

$$f(x) = \begin{cases} x^2 - 4, & 0 \le x < 4 \\ 4, & x = 4 \\ 3x + 0, & 4 < x \end{cases}$$

use algebra to find each of the following limits:

$\lim_{x \to 4^+} f(x) =$ 12

$\lim_{x \to 4^-} f(x) =$ 12

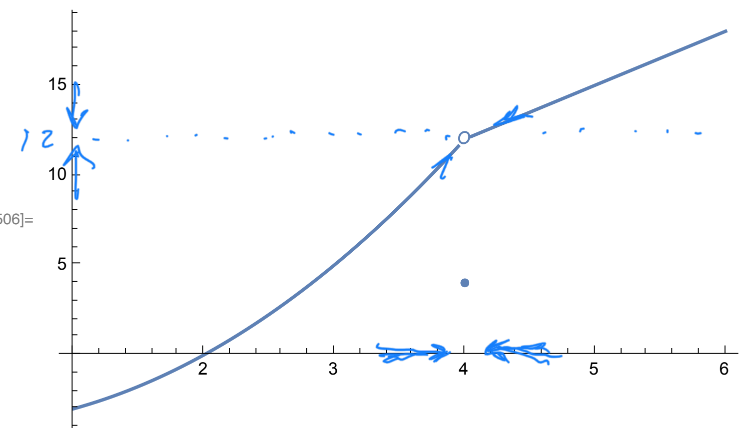
$\lim_{x \to 4} f(x) =$ 12

*(For each, enter **DNE** if the limit does not exist.)*

Sketch a graph of $f(x)$ to confirm your answers.

In[501]:=
```
f[x_] := Piecewise[{{x^2 - 4, 0 ≤ x < 4}, {4, x == 4}, {3 x + 0, 4 < x}}]
p1 = Plot[f[t], {t, 1, 3.95}, PlotStyle → {Thickness[0.005]}];
p1a = Plot[f[t], {t, 4.05, 6}, PlotStyle → {Thickness[0.005]}];
p2 = ListPlot[{{4, 4}}, PlotStyle → Large];
p2a = ListPlot[{{4, 12}}, PlotStyle → Large, PlotMarkers → {O}];
Show[p1, p1a, p2, p2a, PlotRange → {{1, 6}, All}]
```

Out[506]=



The limit is just fine - doesn't care about the hole.

4. Evaluating a limit algebraically.

Evaluate the limit

$$\lim_{x \to -7} \frac{x^2 - 49}{x + 7} =$$

$$\lim_{x \to -7} \frac{(\cancel{x+7})(x-7)}{\cancel{x+7}}$$

If the limit does not exist enter DNE.

$$= \lim_{x \to -7} (x-7)$$

Limit = -14

$$= -14$$

5. Consider the function whose formula is $f(x) = \frac{16-x^4}{x^2-4}$.

a. What is the domain of f? $x^2 - 4 = 0$ when $x = \pm 2$

Domain: $\mathbb{R} - \{-2, 2\}$

b. Use a sequence of values of x near $a = 2$ to estimate the value of $\lim_{x \to 2} f(x)$, if you think the limit exists. If you think the limit doesn't exist, explain why.

c. Use algebra to simplify the expression $\frac{16-x^4}{x^2-4}$ and hence work to evaluate $\lim_{x \to 2} f(x)$ exactly, if it exists, or to explain how your work shows the limit fails to exist. Discuss how your findings compare to your results in (b).

$$f(x) = \frac{16-x^4}{x^2-4} = \frac{(4-x^2)(4+x^2)}{x^2-4} = \frac{-(\cancel{x^2-4})(x^2+4)}{\cancel{x^2-4}} = -(x^2+4)$$

d. True or false: $f(2) = -8$. Why? False: $2 \notin$ Domain

e. True or false: $\frac{16-x^4}{x^2-4} = -4 - x^2$. Why? How is this equality connected to your work above with the function f?

provided $x \neq \pm 2$

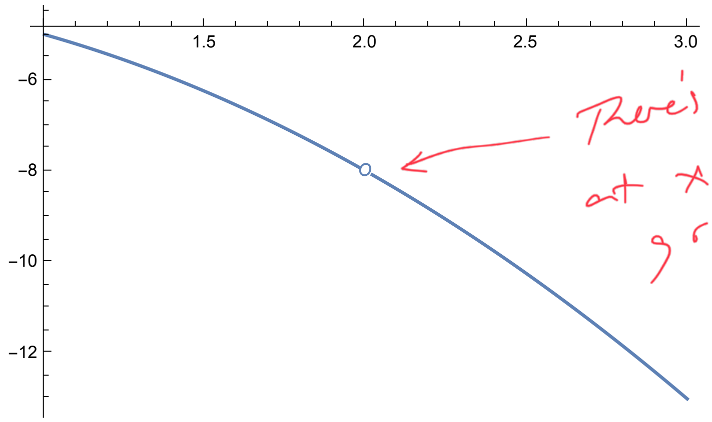
The left hand side & right hand side have different domains.

f. Based on all of your work above, construct an accurate, labeled graph of $y = f(x)$ on the interval $[1, 3]$, and write a sentence that explains what you now know about $\lim_{x \to 2} \frac{16-x^4}{x^2-4}$.

It's going to exist.

In[507]:=
```
f[x_] := (16 - x^4) / (x^2 - 4)
p1 = Plot[f[t], {t, 1, 1.98}, PlotStyle → {Thickness[0.005]}];
p1a = Plot[f[t], {t, 2.02, 3}, PlotStyle → {Thickness[0.005]}];
p2 = ListPlot[{{2, -8}}, PlotStyle → Large, PlotMarkers → {O}];
Show[p1, p1a, p2, PlotRange → {{1, 3}, All}]
```

Out[511]=



There's a hole at $x=2$ in the graph of f – but there's no hole in the graph of $-4-x^2$.

6. Let $g(x) = -\frac{|x+3|}{x+3}$.

a. What is the domain of g? $x+3=0$ when $x=-3$. So $D_g = \mathbb{R} - \{-3\}$

b. Use a sequence of values near $a = -3$ to estimate the value of $\lim_{x\to-3} g(x)$, if you think the limit exists. If you think the limit doesn't exist, explain why. Near $a=-3$, $g(x) = \pm 1$; 1 from the left, -1 from the right.

c. Use algebra to simplify the expression $\frac{|x+3|}{x+3}$ and hence work to evaluate $\lim_{x\to-3} g(x)$ exactly, if it exists, or to explain how your work shows the limit fails to exist. Discuss how your findings compare to your results in (b). (Hint: $|a| = a$ whenever $a \geq 0$, but $|a| = -a$ whenever $a < 0$.)

$$g(x) = \begin{cases} 1 & x < -3 \\ -1 & x > -3 \end{cases}$$

d. True or false: $g(-3) = -1$. Why?

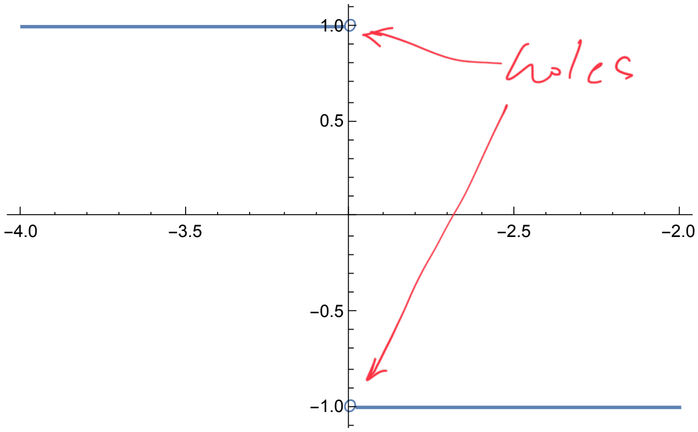
e. True or false: $-\frac{|x+3|}{x+3} = -1$. Why? How is this equality connected to your work above with the function g?

same reasons as above.

f. Based on all of your work above, construct an accurate, labeled graph of $y = g(x)$ on the interval $[-4, -2]$, and write a sentence that explains what you now know about $\lim_{x\to-3} g(x)$.

In[512]:=
```
f[x_] := -Abs[x + 3] / (x + 3)
p1 = Plot[f[t], {t, -4, -3.02}, PlotStyle → {Thickness[0.005]}];
p1a = Plot[f[t], {t, -2.98, -2}, PlotStyle → {Thickness[0.005]}];
p2 = ListPlot[{{-3, -1}, {-3, 1}}, PlotStyle → Large, PlotMarkers → {O}];
Show[p1, p1a, p2, PlotRange → {{-4, -2}, All}]
```

Out[516]=



7. For each of the following prompts, sketch a graph on the provided axes of a function that has the stated properties.

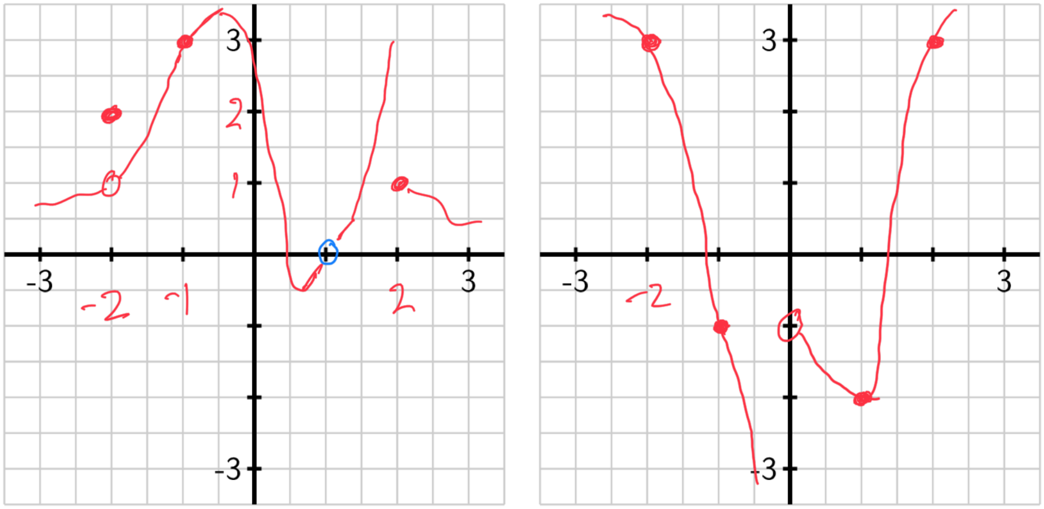


Figure 1.2.12. Axes for plotting $y = f(x)$ in (a) and $y = g(x)$ in (b).

a. $y = f(x)$ such that

- $f(-2) = 2$ and $\lim_{x \to -2} f(x) = 1$
- $f(-1) = 3$ and $\lim_{x \to -1} f(x) = 3$
- $f(1)$ is not defined and $\lim_{x \to 1} f(x) = 0$
- $f(2) = 1$ and $\lim_{x \to 2} f(x)$ does not exist.

b. $y = g(x)$ such that

- $g(-2) = 3$, $g(-1) = -1$, $g(1) = -2$, and $g(2) = 3$
- At $x = -2, -1, 1$ and 2, g has a limit, and its limit equals the value of the function at that point.
- $g(0)$ is not defined and $\lim_{x \to 0} g(x)$ does not exist.