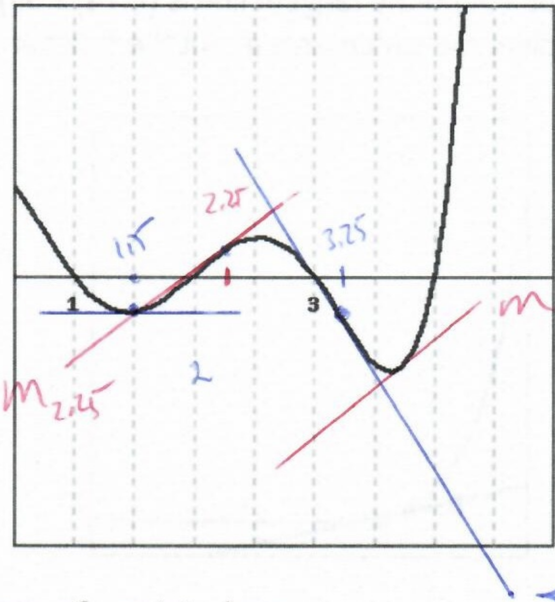
1. Estimating derivative values graphically.

Activate

Consider the function $y = f(x)$ graphed below.

$\to m = 0$



$m_{2.25}$

$m_{2.25}$

The tangent lines are parallel at $x = 2.25$ & $x = 3.75$

Slope $m < 0$

Give the x-coordinate of a point where:

A. the derivative of the function is negative: $x =$ 3.25

B. the value of the function is negative: $x =$ 1.5

C. the derivative of the function is smallest (most negative): $x =$ 3.25

D. the derivative of the function is zero: $x =$ 1.5

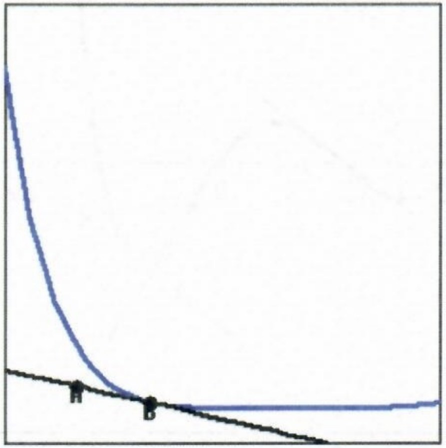
The steepest downward slanting tangent

E. the derivative of the function is approximately the same as the derivative at $x = 2.25$ (be sure that you give a point that is distinct from $x = 2.25$!): $x =$ 3.75

2. Tangent line to a curve.

Activate

The figure below shows a function $g(x)$ and its tangent line at the point $B = (6.8, 2)$. If the point A on the tangent line is $(6.74, 2.05)$, fill in the blanks below to complete the statements about the function g at the point B.



$g($ 6.8 $) =$ 2

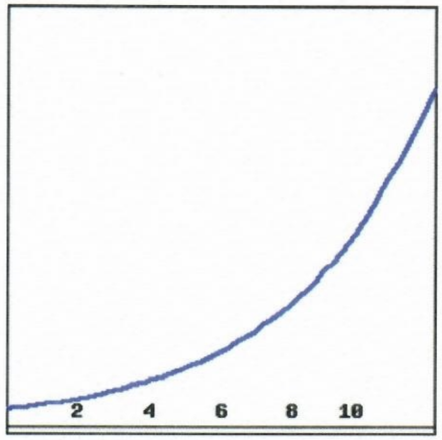
$g'($ 6.8 $) =$ $-5/6$

$$\frac{2.05 - 2}{6.74 - 6.8} = \frac{0.05}{-0.06} = -5/6$$

3. Interpreting values and slopes from a graph.

Activate

Consider the graph of the function $f(x)$ shown below.



Using this graph, for each of the following pairs of numbers decide which is larger. *Be sure that you can explain your answer.*

A. $f(6) < f(8)$ (function is increasing)

B. $f(6)-f(4) > f(4)-f(2)$ (Think slopes of secant lines - Just divide each side by 2)

C. $\dfrac{f(4)-f(2)}{4-2} < \dfrac{f(6)-f(4)}{6-4}$

As is done here!

D. $f'(2) < f'(8)$ (Tangent lines get steeper as x increases)

4. Finding an exact derivative value algebraically.

Activate

Find the derivative of $g(t) = 2t^2 + 2t$ at $t = 7$ algebraically.

$g'(7) =$ 30

$$g'(7) = \lim_{h \to 0} \frac{g(7+h) - g(7)}{h}$$

$$= \lim_{h \to 0} \frac{2(7+h)^2 + 2(7+h) - (2 \cdot 7^2 + 2 \cdot 7)}{h}$$

$$= \lim_{h \to 0} \frac{2(\cancel{7^2} + 2h7 + h^2) + \cancel{2 \cdot 7} + 2 \cdot h - \cancel{2 \cdot 7^2} + \cancel{2 \cdot 7}}{h}$$

$$= \lim_{h \to 0} \frac{28h + 2h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{30h + 2h^2}{h}$$

polynomial in h - replace h with 0:

$$= \lim_{h \to 0} \frac{\cancel{h}(30 + 2h)}{\cancel{h}} = \lim_{h \to 0} (30 + 2h) = 30$$

5. Estimating a derivative from the limit definition.

Activate

Estimate $f'(3)$ for $f(x) = 6^x$. Be sure your answer is accurate to within 0.1 of the actual value.

$f'(3) \approx$ 387

$$\frac{6^{3+.0001} - 6^3}{3+.0001 - 3} \approx 387.055$$

True values in here

$$\frac{6^{3-.0001} - 6^3}{3-.0001-3} \approx 386.985$$

within 0.01

Tangent, $x=-3$

secant $-3,-1$

$m \approx \frac{3.5}{3} \approx 1.17$

6. Consider the graph of $y = f(x)$ provided in Figure 1.3.12.

a. On the graph of $y = f(x)$, sketch and label the following quantities:

- the secant line to $y = f(x)$ on the interval $[-3, -1]$ and the secant line to $y = f(x)$ on the interval $[0, 2]$.
- the tangent line to $y = f(x)$ at $x = -3$ and the tangent line to $y = f(x)$ at $x = 0$.

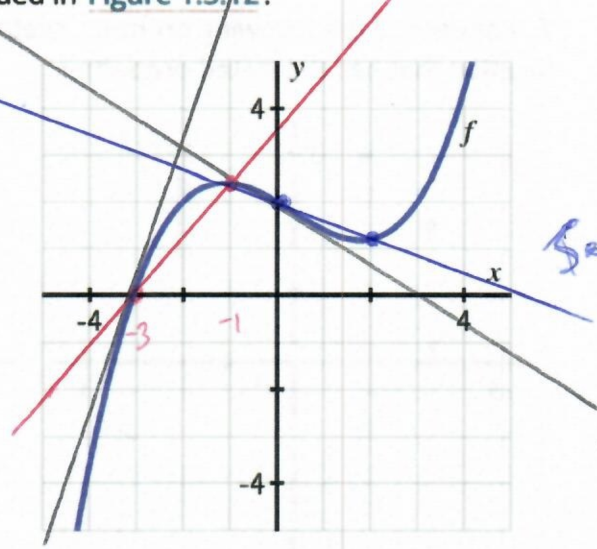


Figure 1.3.12. Plot of $y = f(x)$.

Secant, 0, 2

$m \approx \frac{-2}{6}$

$\approx -\frac{1}{3}$

Tangent

$x=0$

b. What is the approximate value of the average rate of change of f on $[-3, -1]$? On $[0, 2]$? How are these values related to your work in (a)?

slopes of secant lines

on $[-3, 1]$

$AV \approx 1.17$

on $[0, 2]$

$AV \approx -\frac{1}{3}$

c. What is the approximate value of the instantaneous rate of change of f at $x = -3$? At $x = 0$? How are these values related to your work in (a)?

Slopes of tangent lines

$f'(-3) \approx 3.5$ (using "points" $(-3,0)$ & $(-2,3.5)$)

$f'(0) \approx -\frac{2}{3}$ (using $(0,2)$ & $(3,0)$)

7. For each of the following prompts, sketch a graph on the provided axes in Figure 1.3.13 of a function that has the stated properties.

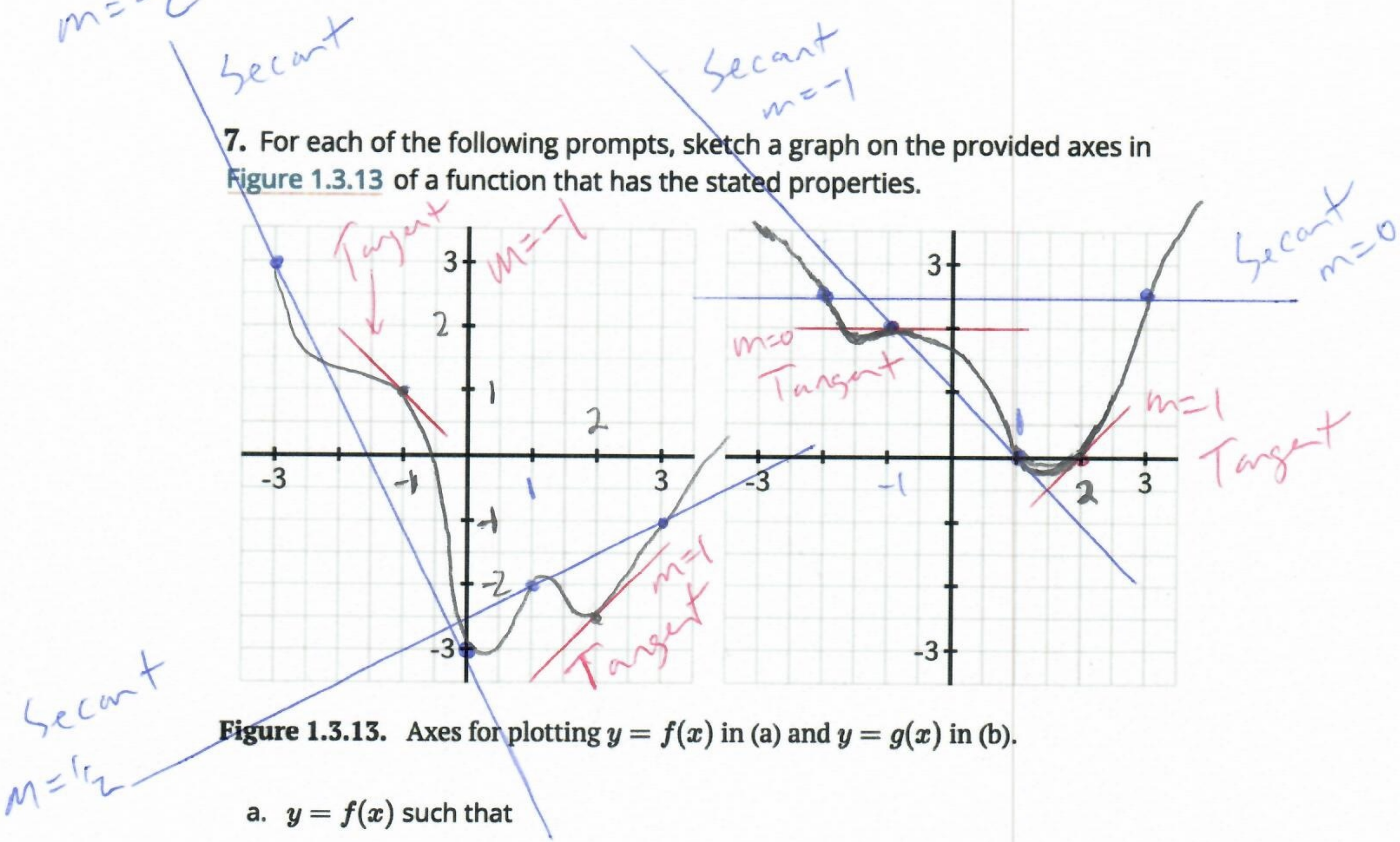


Figure 1.3.13. Axes for plotting $y = f(x)$ in (a) and $y = g(x)$ in (b).

a. $y = f(x)$ such that

- the average rate of change of f on $[-3, 0]$ is -2 and the average rate of change of f on $[1, 3]$ is 0.5, and
- the instantaneous rate of change of f at $x = -1$ is -1 and the instantaneous rate of change of f at $x = 2$ is 1.

b. $y = g(x)$ such that

- $\frac{g(3)-g(-2)}{5} = 0$ and $\frac{g(1)-g(-1)}{2} = -1$, and
- $g'(2) = 1$ and $g'(-1) = 0$