domain of g', $g'(x) = \frac{1}{f'(g(x))}$.

Chain rule: $\ln(\text{stuff})$. $\leftarrow t^3+3$

$\frac{d}{dx}\ln(x) = \frac{1}{x}$.

$\therefore$

2.6.6 Exercises

1. Composite function involving logarithms and polynomials.

Activate

Find the derivative of the function $f(t)$, below.

$f(t) = \ln(t^3 + 3)$

$f'(t) =$ $f'(t) = \frac{1}{\text{stuff}} \cdot \text{stuff}' = \frac{1}{t^3+3} \cdot 3t^2 = \frac{3t^2}{t^3+3}$

2. Composite function involving trigonometric functions and logarithms.

Activate

$g(t) = \cos(\text{stuff})$, $\text{stuff} = \ln(t)$

$\frac{d}{dx}\cos(x) = -\sin(x)$.

Find the derivative of the function $g(t)$, below. It may be to your advantage to simplify before differentiating.

$g(t) = \cos(\ln(t))$

$g'(t) =$ $-\sin(\text{stuff}) \cdot \text{stuff}' = -\sin(\ln(t)) \cdot \frac{1}{t} = \frac{-\sin(\ln(t))}{t}$

3. Product involving $\arcsin(w)$.

Activate

Just a product rule:

Find the derivative of the function $h(w)$, below. It may be to your advantage to simplify before differentiating.

$h(w) = 7w \arcsin w$

$h'(w) =$ $(7w)' \cdot \arcsin(w) + 7w\,(\arcsin'(w)) = 7\arcsin(w) + \frac{7w}{\sqrt{1-w^2}}$

4. Derivative involving $\arctan(x)$.

$\arctan'(x) = \frac{1}{1+x^2}$

$\frac{1}{x}$ whose derivative is $\frac{-1}{x^2}$

$f(x) = \underbrace{\arctan(x) + \arctan(\text{stuff})}_{\text{sum rule.}}$

$(\arctan(\text{stuff}))' = \frac{1}{1+\text{stuff}^2} \cdot \text{stuff}'$

Activate

For $x > 0$, find ***and simplify*** the derivative of
$f(x) = \arctan x + \arctan(1/x)$.

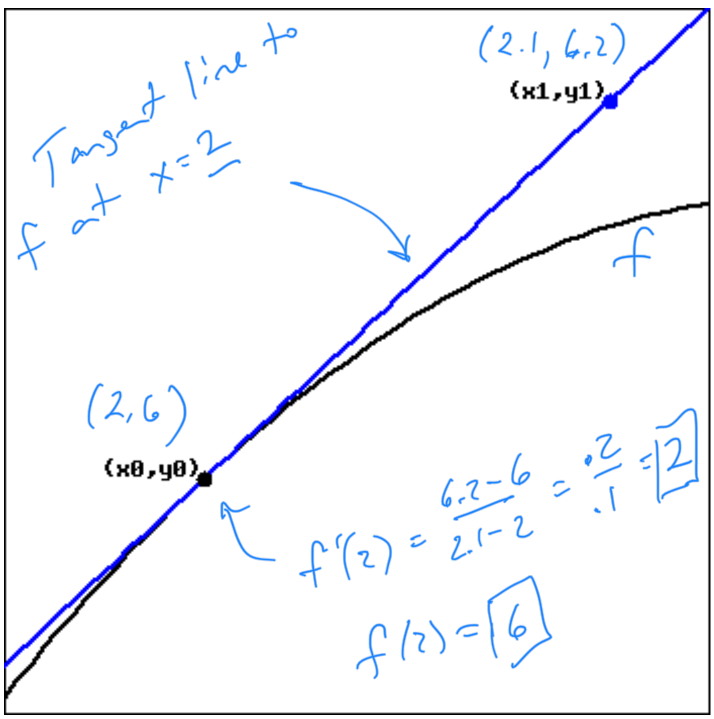
$f'(x) =$ $\frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \cdot \left(\frac{-1}{x^2}\right) = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0!$

(What does your result tell you about f)? Its derivative is 0 everywhere (except at $x=0$, where it's not defined). Constant (but graph f - it's interesting!)

5. Composite function from a graph.

Activate

Let $(x_0, y_0) = (2, 6)$ and $(x_1, y_1) = (2.1, 6.2)$. Use the following graph of the function f to find the indicated derivatives.



Note: $g = f^{-1}$.
If $f(2) = 6$, then
$g(6) = 2$
One undoes the other!

If $h(x) = (f(x))^5$, then $h'(x) = 5(f(x))^4 \cdot f'(x)$

$h'(2) =$ $5(f(2))^4 \cdot f'(2) = 5(6)^4 \cdot 2 = 12960$

If $g(x) = f^{-1}(x)$, then $(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$, so

$g'(6) =$ $\frac{1}{f'(g(6))} = \frac{1}{f'(2)} = \frac{1}{2}$

6. Composite function involving an inverse trigonometric function.

$$\left(\sin^{-1}(\text{stuff})\right)' = \frac{1}{\sqrt{1-\text{stuff}^2}} \cdot \text{stuff}'$$

Activate

Let

$$f(x) = 7\sin^{-1}(x^3)$$

$f'(x) =$ $\frac{7}{\sqrt{1-(x^3)^2}} \cdot (3x^2) = \frac{21x^2}{\sqrt{1-x^6}}$

NOTE: The webwork system will accept $\arcsin(x)$ or $\sin^{-1}(x)$ as the inverse of $\sin(x)$.

$$\left(\arctan(\overset{\text{stuff}}{3x^3})\right)' = \frac{1}{1+(3x^3)^2} \cdot (3x^3)'$$

7. Mixing rules: product, chain, and inverse trig.

Activate

If $f(x) = \overbrace{8x^4\arctan(3x^3)}^{\text{product}}$, find $f'(x)$.

$f'(x) =$ $32x^3 \cdot \arctan(3x^3) + 8x^4 \cdot \frac{1}{1+9x^6} \cdot 9x^2 =$

$8x^3\left(\arctan(3x^3) + \frac{9x^3}{1+9x^6}\right)$

8. Mixing rules: product and inverse trig.

Activate

Let $f(x) = 8\cos(x)\sin^{-1}(x)$. Find $f'(x)$.

$f'(x) =$ $8\sin(x) \cdot \arcsin(x) + 8\cos(x)\frac{1}{\sqrt{1-x^2}}$

9. Determine the derivative of each of the following functions. Use proper notation and clearly identify the derivative rules you use.

They all look us'g! 😊

a. $f(x) = \ln(2\arctan(x) + 3\arcsin(x) + 5)$

b. $r(z) = \arctan(\ln(\arcsin(z)))$

c. $q(t) = \arctan^2(3t)\arcsin^4(7t)$

d. $g(v) = \ln\left(\dfrac{\arctan(v)}{\arcsin(v) + v^2}\right)$

d down below; at 12

a. ln(stuff). $f'(x) = \frac{1}{2\arctan(x) + 3\arcsin(x) + 5} \cdot \left(\frac{2}{1+x^2} + \frac{3}{\sqrt{1-x^2}} + 0\right)$

sum rule

b, Triple composition ; arctan(stuff).

$\frac{1}{1+(\ln(\arcsin(z)))^2} \cdot (\underset{\ln(\text{stuff})}{\ln(\arcsin(z))})'$

$= \frac{1}{1+(\ln(\arcsin(z)))^2} \cdot \frac{1}{\arcsin(z)} \cdot \arcsin'(z)$

$= \frac{1}{1+(\ln(\arcsin(z)))^2} \cdot \frac{1}{\arcsin(z)} \cdot \frac{1}{\sqrt{1-x^2}}$

10. Consider the graph of $y = f(x)$ provided in Figure 2.6.7 and use it to answer the following questions.

c. Product of triple compositions:

$q'(t) = 2(\arctan(3t)) \cdot (\arctan(3t))' \cdot (\arcsin(7t))^4 +$
$(\arctan(3t))^2 \cdot 4(\arcsin(7t))^3 \cdot (\arcsin(7t))'$

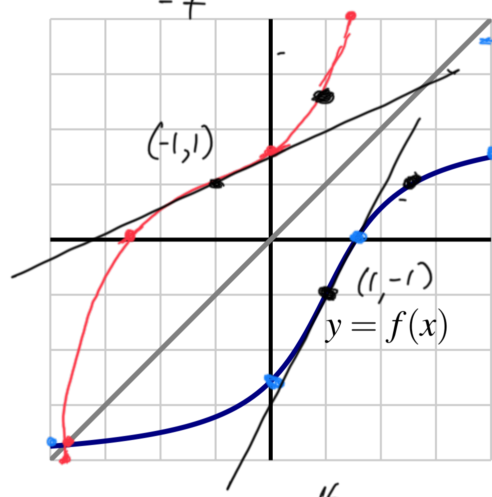
$= 2(\arctan(3t)) \cdot \frac{1}{1+(3t)^2} \cdot (3t)' \cdot (\arcsin(7t))^4 +$

$4(\arctan(3t))^2 \cdot (\arcsin(7t))^3 \cdot \frac{1}{\sqrt{1-(7t)^2}} \cdot \underbrace{(7t)'}_{=7}$ & then simplify

a. Use the provided graph to estimate the value of $f'(1)$. $= 3$

b. Sketch an approximate graph of $y = f^{-1}(x)$. Label at least three distinct points on the graph that correspond to three points on the graph of f. $m = \frac{1}{2}$

c. Based on your work in (a), what is the value of $(f^{-1})'(-1)$? Why?



mirror $y=x$

every point is reflected in the mirror

$m \approx 4/2 = 2$

Slopes of reflected curves at reflected points are ~~additive~~ multiplicative inverses.

Figure 2.6.7. A function $y = f(x)$

11. Let $f(x) = \frac{1}{4}x^3 + 4$.

a. Sketch a graph of $y = f(x)$ and explain why f is an invertible function.

b. Let g be the inverse of f and determine a formula for g.

c. Compute $f'(x)$, $g'(x)$, $f'(2)$, and $g'(6)$. What is the special relationship between $f'(2)$ and $g'(6)$? Why?

12. Let $h(x) = x + \sin(x)$.

a. Sketch a graph of $y = h(x)$ and explain why h must be invertible.

b. Explain why it does not appear to be algebraically possible to determine a formula for h^{-1}.

c. Observe that the point $(\frac{\pi}{2}, \frac{\pi}{2} + 1)$ lies on the graph of $y = h(x)$. Determine the value of $(h^{-1})'(\frac{\pi}{2} + 1)$.

9d. $g(v) = \ln\left(\frac{\arctan(v)}{\arcsin(v) + v^2}\right) = \ln(\text{stuff})$.

quotient, use Q Rule:

$g'(v) = \frac{1}{\left(\frac{\arctan(v)}{\arcsin(v)+v^2}\right)} \cdot \left(\frac{\arctan(v)}{\arcsin(v)+v^2}\right)'$ $\frac{\frac{1}{1+v^2}\cdot(\arcsin(v)+v^2) - \arctan(v)\cdot\left(\frac{1}{\sqrt{1-v^2}}+2v\right)}{(\arcsin(v)+v^2)^2}$

$= \frac{\arcsin(v) + v^2}{\arctan(v)} \cdot$

= one gnarly mess?

Feedback

$$g'(v) = \frac{\dfrac{\arcsin(v)+v^2}{1+v^2} - \arctan(v)\left(\dfrac{1}{\sqrt{1-v^2}} + 2v\right)}{\arctan(v)\left(\arcsin(v)+v^2\right)}$$

??

$$= \underbrace{\frac{1}{\arctan(v)\left(1+v^2\right)} - \frac{1+2v\sqrt{1-v^2}}{\sqrt{1-v^2}\left(\arcsin(v)+v^2\right)}}$$

Is this better, or worse?!

One tries to "simplify" until the expression is as enlightening as possible – but with this ugly function, nothing may enlighten us! :)