Section 3.5 Homework

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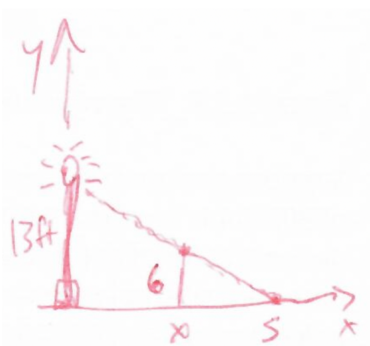
Problems 2-6

2. Movement of a shadow.

Activate

A street light is at the top of a 13 foot tall pole. A 6 foot tall woman walks away from the pole with a speed of 6 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 30 feet from the base of the pole?

The tip of the shadow is moving at ________________ ft/sec.



Observations:

We want to relate dx/dt and ds/dt.

By similar triangles, (s-x)/6=s/13

s=(13/7)x

ds/dt=(13/7)dx/dt

ds/dt=(13/7)6 ft/s = 78/7 ft/s

3. A leaking conical tank.

Activate

Water is leaking out of an inverted conical tank at a rate of 9600.0 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 7.0 m and the the diameter at the top is 5.0 m. If the water level is rising at a rate of 22.0 cm/min when the height of the water is 1.5 m, find the rate at which water is being pumped into the tank in cubic centimeters per minute.

Answer: ______________________ cm^3/min

Observations:

volume(h,r)=1/3 pi r^2 h

dVin/dt = $9600 + \frac{5^2\pi}{14^2} 150^2 \cdot 22$

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In[1313]:= 9600 + 5^2 Pi / 14^2 * 150^2 * 22
N[%]
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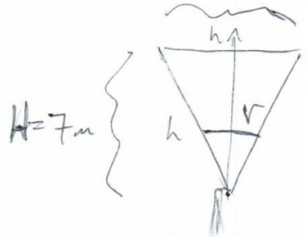
Out[1313]= $9600 + \frac{3\,093\,750\,\pi}{49}$

Out[1314]= 207 953.107592723

\#3 $\qquad D = 5m = 2R$ $\qquad$ (1)



$$\frac{dV_{out}}{dt} = -9600 \ \mathrm{cm^3/min}$$

$$\frac{dV_{in}}{dt} = \text{constant.}$$

$$\left.\frac{dh}{dt}\right|_{1.5m} = 22 \ \mathrm{cm/min}$$

Relate $\frac{dh}{dt}$ & $\frac{dV_{in}}{dt}$

$$\frac{dV}{dt} = \frac{dV_{out}}{dt} + \frac{dV_{in}}{dt} = -9600 + \frac{dV_{in}}{dt}$$

$$V = \frac{1}{3}\pi r^2 h$$

Similar triangles: $\frac{r}{h} = \frac{R}{H} \rightarrow r = \frac{R}{H}h = \frac{5/2}{7}h = \frac{5}{14}h$

$$\therefore V = \frac{1}{3}\pi\left(\frac{5}{14}h\right)^2 h = \underbrace{\frac{5^2\pi}{3(14)^2}}_{c} h^3$$

$$\boxed{\frac{dV}{dt} = c\, 3h^2 \frac{dh}{dt}}$$

we've related the rates!

$$\frac{dV_{in}}{dt} - 9600 = 3c\, \underbrace{h^2 \frac{dh}{dt}}_{\text{we know these}}$$

when $h = 1.5m = 150 \ \mathrm{cm}$

$\frac{dh}{dt} = 22 \ \mathrm{cm/min}$

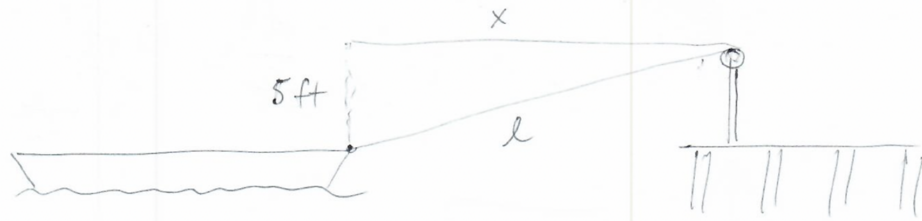
$$\frac{dV_{in}}{dt} = 9600 + \frac{5^2\pi}{14^2}(150)^2 \cdot 22 \approx 208000 \ \mathrm{cm^3/min}$$

Out[1305]= $9600 + \frac{3\,093\,750\,\pi}{49}$

Out[1306]= 207 953.107592723

4. A sailboat is sitting at rest near its dock. A rope attached to the bow of the boat is drawn in over a pulley that stands on a post on the end of the dock that is 5 feet higher than the bow. If the rope is being pulled in at a rate of 2 feet per second, how fast is the boat approaching the dock when the length of rope from bow to pulley is 13 feet?

#4



$$\frac{dl}{dt} = -2 \text{ ft/s}$$

Rate at which boat approaches dock $= \frac{dx}{dt}$

Relate those rates! Easy:

$$5^2 + x^2 = l^2$$

Differentiate both sides:

$$0 + 2x\frac{dx}{dt} = 2l\frac{dl}{dt}$$

when $l = 13$ ft, $x = \sqrt{13^2 - 5^2} = 12$ ft

& $\frac{dl}{dt} = -2$.

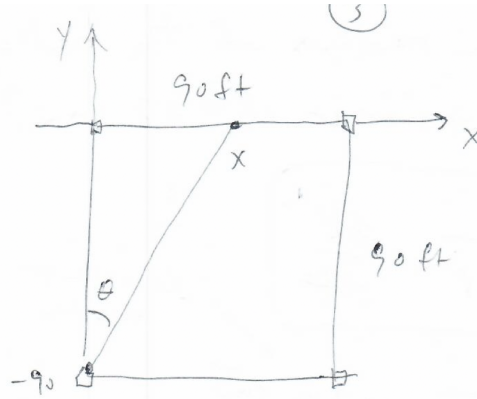
$\therefore\ 24\frac{dx}{dt} = -4 \cdot 13$ $\qquad \boxed{\frac{dx}{dt} = \frac{-13}{6} \text{ ft/s}}$

$\simeq 2.2$ ft/s

6. A baseball diamond is a square with sides 90 feet long. Suppose a baseball player is advancing from second to third base at the rate of 24 feet per second, and an umpire is standing on home plate. Let θ be the angle between the third baseline and the line of sight from the umpire to the runner. How fast is θ changing when the runner is 30 feet from third base?

3

#6



$$\frac{dx}{dt} = -24 \text{ ft/s}$$

(x is decreasing, heading for 0).

Relate to $\frac{d\theta}{dt}$

$$\tan\theta = \frac{x}{90}$$

Differentiate both sides!

$$\frac{1}{1+\theta^2}\frac{d\theta}{dt} = \frac{1}{90}\frac{dx}{dt} = \frac{-24}{90} \text{ ft/s}$$

$$\frac{d\theta}{dt} = \frac{-4}{15}(1+\theta^2)$$

When $x = 30$ ft,

$$\frac{d\theta}{dt} = \frac{-4}{15}\left(1+(\arctan 1/3)^2\right)$$

$$\approx -.294 \text{ radians/s}$$

In[1315]:=
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-4 / 15 (1 + (ArcTan[1 / 3]) ^2)
N[%]
Plot[-4 / 15 (1 + (ArcTan[x / 90]) ^2), {x, 0, 90}]
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Out[1315]= $-\frac{4}{15}\left(1+\text{ArcTan}\left[\frac{1}{3}\right]^2\right)$

Out[1316]= -0.294272911801212

Out[1317]=