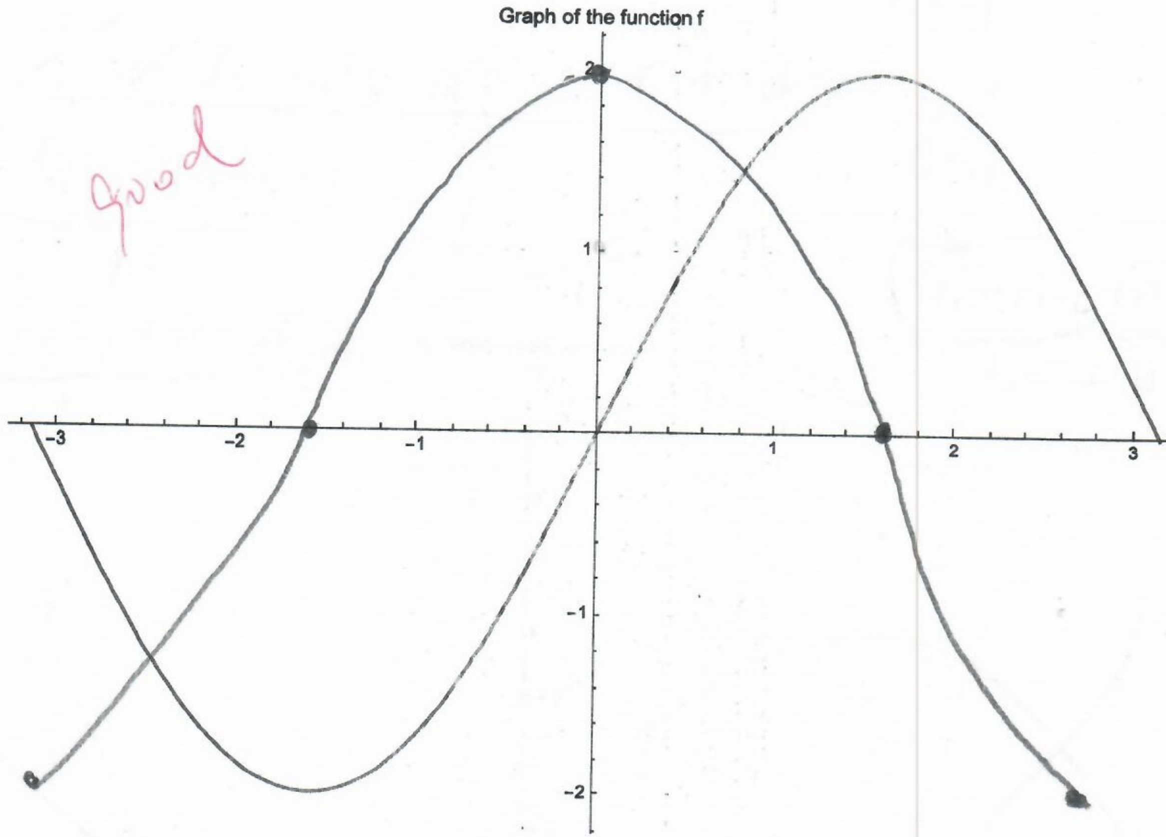
1. (4 pts) Carefully (i.e. using estimates from tangent lines) plot the derivative function f'(x) on the following plot of f:




You may invoke any properties of f to simplify your work.

Might mention symmetry

2. (a) (4 pts) Consider the function $f(x) = 1 - 4x + x^3$. Below you see its graph. Use the limit definition to find an algebraic expression for the derivative function f'(x).

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

$\lim\limits_{h\to 0}$ ! It's the "limit definition" after all....

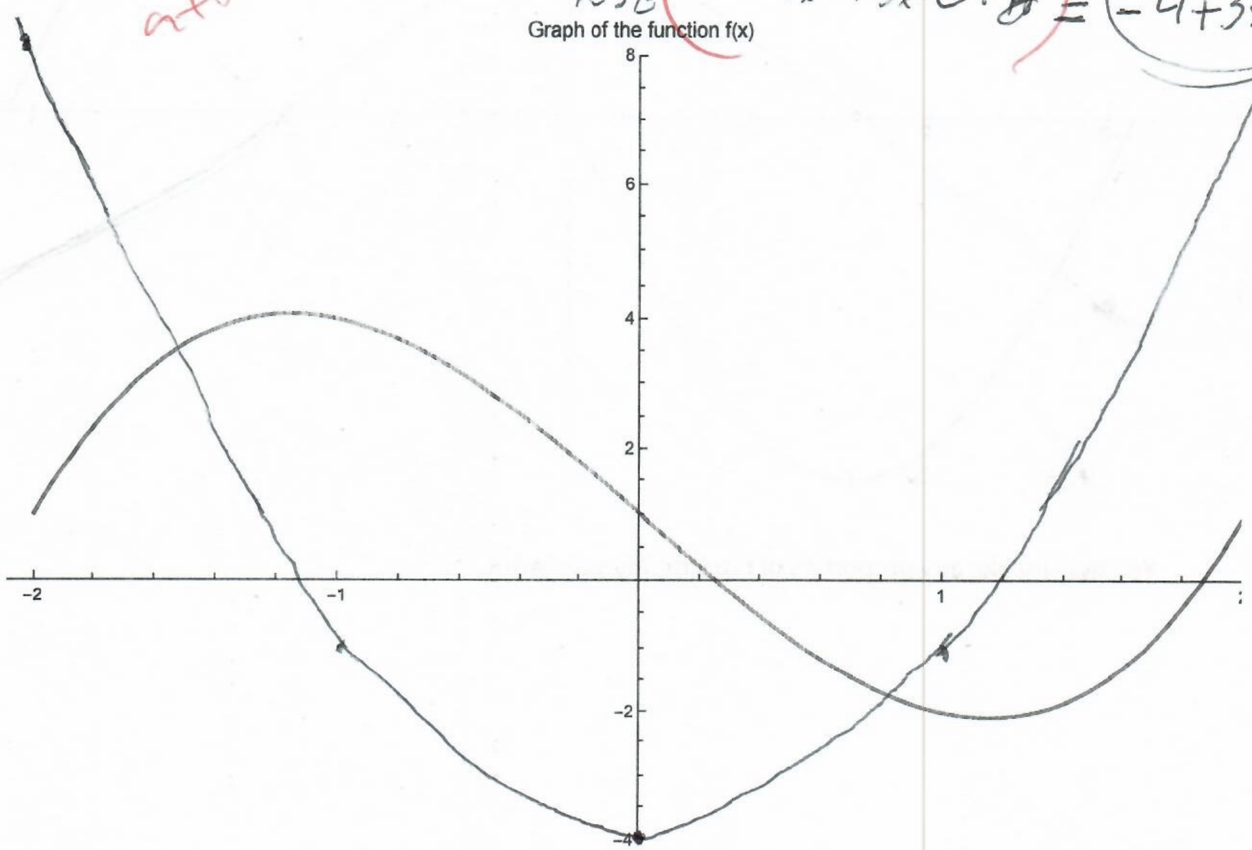
$$\lim_{h\to 0} \frac{(1-4(x+h)+(x+h)^3 - (1-4(x)+(x)^3)}{h}$$

$$\lim_{h\to 0} \frac{(1-4x-4h+(x+h)^3 - 1+4x+x^3}{h}$$

$$\lim_{h\to 0} \frac{-4h+x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(-4+3x^2+3xh+h^2)}{\cancel{h}}$$

$$\lim_{h\to 0} \left(-4+3x^2+3xh+h^2\right) = -4+3x^2$$




2. (b) (2 pts) Add the graph of f'(x) onto the graph above, and explain why the derivative makes sense -- how does it correspond to the behavior of f?

good

The value of the slopes are positive until around -1.25 + 1.25 where the reach zero. This corresponds to the slope of the tangent line being positive before -1.25 and after 1.25 and in those numbers the slope is a turn dips to - numbers

$f(x)=1-4x+x^3$
$=x^3-4x+1$

2. (a) (4 pts) Consider the function $f(x) = 1 - 4x + x^3$. Below you see its graph. Use the limit definition to find an algebraic expression for the derivative function f'(x).

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{((x+h)^3-4(x+h)+1-(x^3-4x+1))}{h}$$

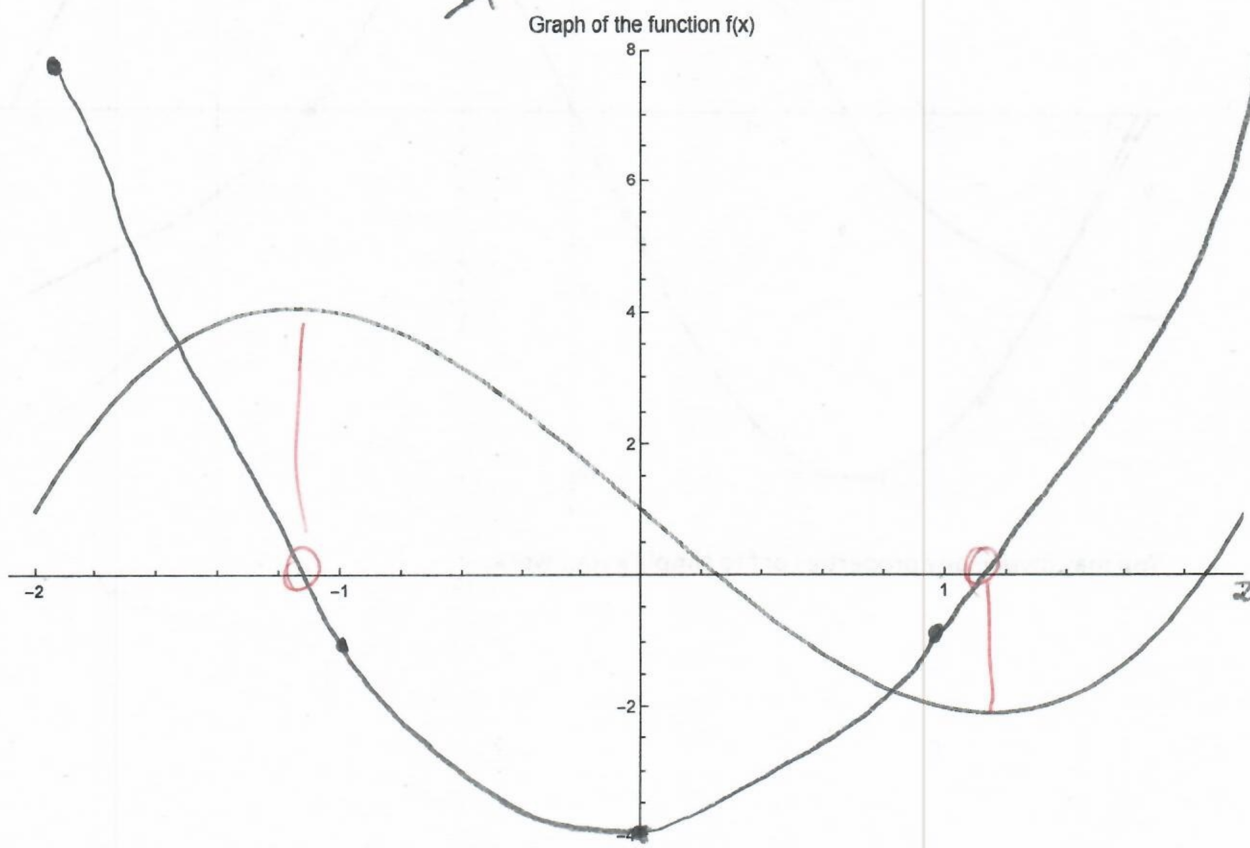
$$=\lim_{h\to 0}\frac{(\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{4x}-4h+\cancel{1})-(\cancel{x^3}-\cancel{4x}+\cancel{1})}{h}$$

$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-4h}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2-4)}{\cancel{h}}$$

$$=\lim_{h\to 0}(3x^2+\cancel{3xh}+\cancel{h^2}-4)$$

$$\boxed{f'(x)=3x^2-4}$$




2. (b) (2 pts) Add the graph of f'(x) onto the graph above, and explain why the derivative makes sense -- how does it correspond to the behavior of f?

| X | Y |
|---|---|
| -2 | 8 |
| -1 | -1 |
| 0 | -4 |
| 1 | -1 |
| 2 | 8 |

The derivative is a quadratic function, which makes sense because f(x) is a 3rd degree polynomial.

$(x^2+2xh+h^2)(x+h)$

$x^3+x^2h+2x^2h+2xh^2+xh^2+h^3$

$x^3+3x^2h+3xh^2+h^3$

$x^3-4x+1$

2. (a) (4 pts) Consider the function $f(x) = 1 - 4x + x^3$. Below you see its graph. Use the limit definition to find an algebraic expression for the derivative function f'(x).

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

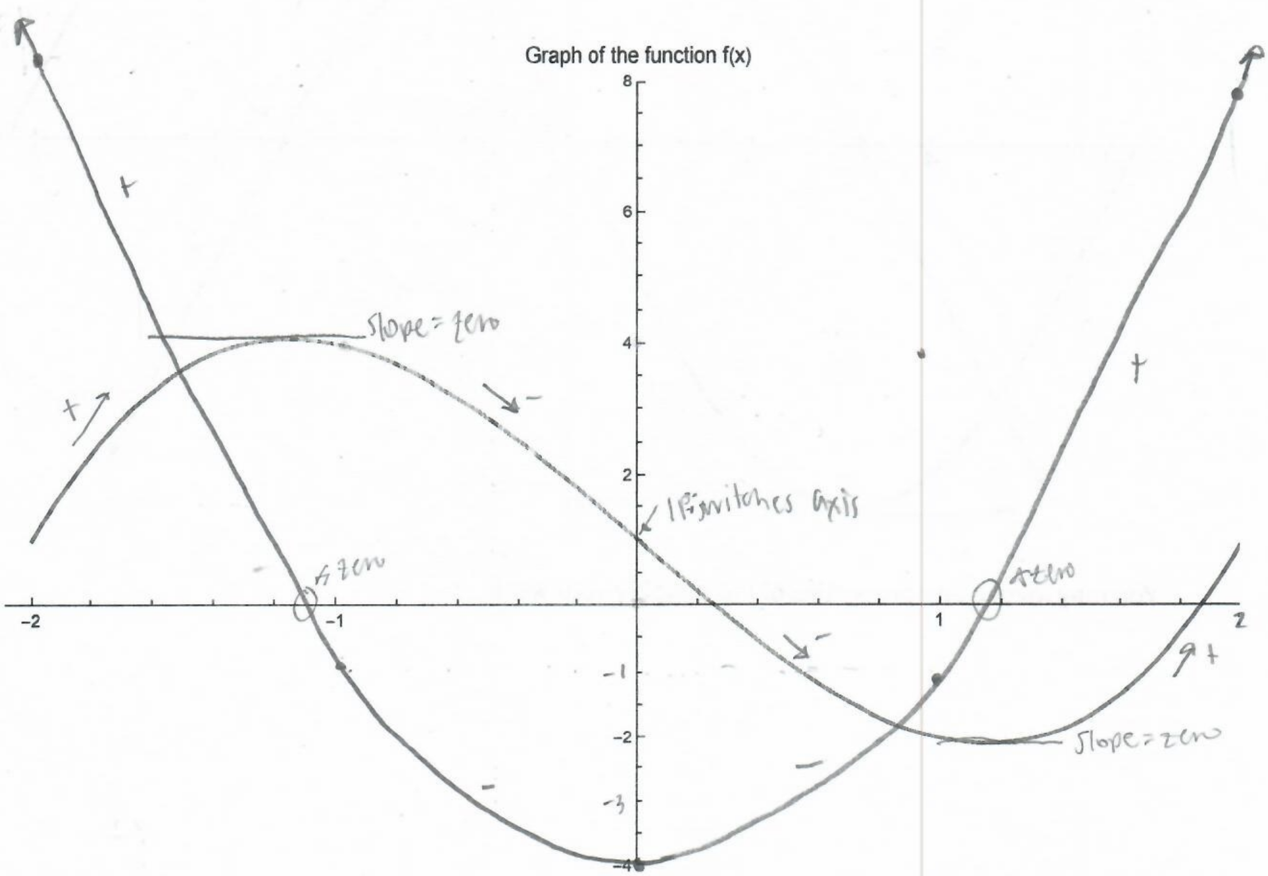
$$= \lim_{h\to 0} \frac{[(x+h)^3-4(x+h)+1]-[x^3-4x+1]}{h}$$

$$= \lim_{h\to 0} \frac{[\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{4x}-4h+\cancel{1}]-[\cancel{x^3}-\cancel{4x}+\cancel{1}]}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-4h}{h}$$

$$= \lim_{h\to 0} (3x^2+3xh+h^2-4)$$

$$\boxed{f'(x) = 3x^2-4}$$




2. (b) (2 pts) Add the graph of f'(x) onto the graph above, and explain why the derivative makes sense -- how does it correspond to the behavior of f?

the derivative makes sense because when the slope = 0, the y value of the derivative is 0. Also, at the inflection point, the graph switches its axis.