Quiz 10, Spring 2024

Name:

7. Let p be a function whose second derivative is $p''(x) = (x+1)(x-2)e^{-x}$.

a. Construct a second derivative sign chart for p and determine all inflection points of p.

b. Suppose you also know that $x = \frac{\sqrt{5}-1}{2}$ is a critical number of p. Does p have a local minimum, local maximum, or neither at $x = \frac{\sqrt{5}-1}{2}$? Why?

c. If the point $(2, \frac{12}{e^2})$ lies on the graph of $y = p(x)$ and $p'(2) = -\frac{5}{e^2}$, find the equation of the tangent line to $y = p(x)$ at the point where $x = 2$. Does the tangent line lie above the curve, below the curve, or neither at this value? Why?

x	$-\infty$		-1	$\frac{\sqrt{5}-1}{2}$	2		∞
$p''(x)$	∞	$+$	0	$-$	0	$+$	0
$p'(x)$				0			
$p(x)$					$\frac{12}{e^2}$		

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In[677]:= Clear[x]
fpp[x_] := (x + 1) (x - 2) E^(-x)
fp[x_] = Integrate[fpp[x], x]
f[x_] = Integrate[fp[x], x]
tangent[x_] = f[2] + fp[2] (x - 2)
f''[2]
Plot[{f[x], fp[x], fpp[x], tangent[x]}, {x, -3, 6}]
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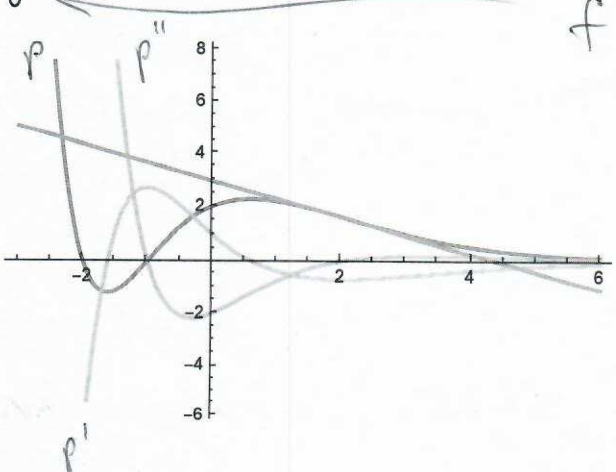
Out[679]= $-e^{-x}\left(-1 + x + x^2\right)$

Out[680]= $-e^{-x}\left(-2 - 3x - x^2\right)$

Out[681]= $\frac{12}{e^2} - \frac{5(-2+x)}{e^2}$ ← Tangent line

Out[682]= 0 ← $f''(2) = 0 \rightarrow$ inflection point

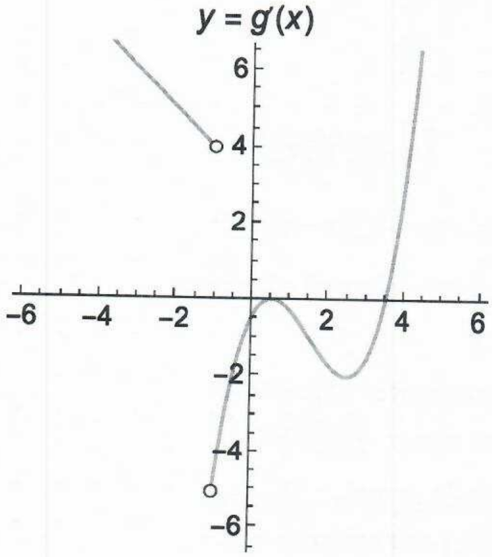
Out[683]=



line is neither above nor below

1. Function $g(x)$ has domain $(-\infty, \infty)$. The graph shown below is of the derivative $g'(x)$ (NOT $g(x)$). However, use the graph to answer questions about the original function $g(x)$.

Out[695]=



x	$-\infty$		-1		0.5		3.5		∞
$g'(x)$	∞	$+$	$4 \| -5$	$-$	0	$-$	0	$+$	∞
$g(x)$		↗	?	↘	•	↘	−	↗	

inflection point

1.1. What are the intervals of increase for $g(x)$. (What does that mean for $g'(x)$?)

$(-\infty, -1) \cup (3.5, \infty) \rightarrow g'(x) > 0$

1.2. What are the intervals of decrease for $g(x)$. (What does that mean for $g'(x)$?)

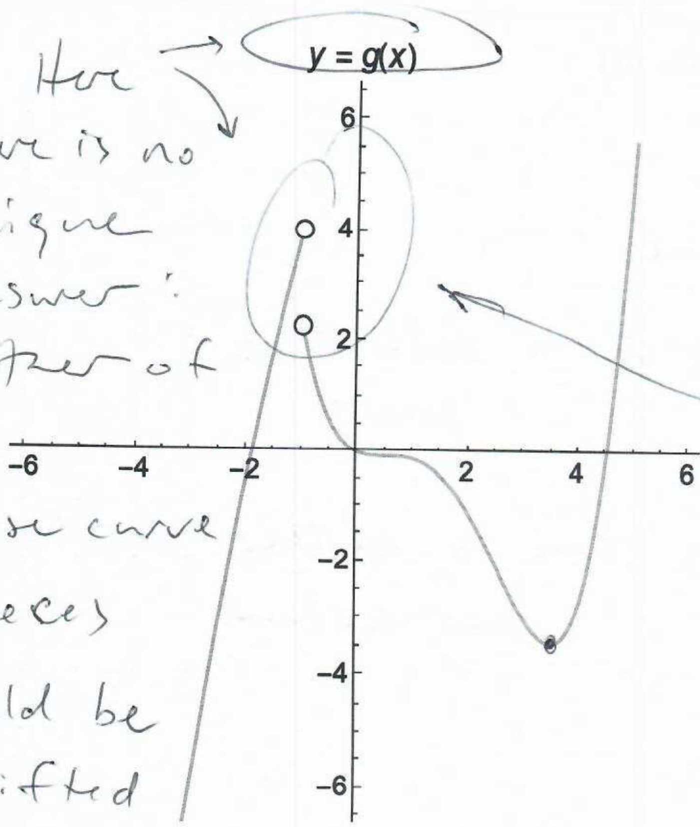
$(-1, 0.5) \cup (0.5, 3.5) \longrightarrow g'(x) < 0$

1.3. Find all the critical points for $g(x)$, and for each one determine if it is a local maximum, a local minimum, or neither.

Here

there is no unique answer: either of those curve pieces could be shifted up or down w/o affecting the derivative

Out[710]=



Critical Pt	Extremum
$x = 0.5$	inflection
$x = -1$	unsure: If the two curves are shifted so as to join, we may have a local max.
$x = 3.5$	local ~~maximum~~ minimum!

$-\ \ 0\ \ +$

↘ − ↗