Section 2.1 and 2.3 Worksheet

Power and Linear Rules Worksheet

1. Let $f(x) = 2x^3 - 15x^2 + 24x - 10$.

In[729]:=
```
f[x_] := 2 x^3 - 15 x^2 + 24 x - 10
```

1.1. Compute the derivative $f'(x)$.

In[730]:=
```
fp[x_] := f'[x]
fp[x]
```

Out[731]= $24 - 30x + 6x^2$

1.2. What is the slope of the tangent line when $x = 0$?

In[732]:=
```
f'[0]
```

Out[732]= 24

1.3. What are the x-values for points on $y = f(x)$ where the slope of the tangent line is 0? (This is NOT the same question as 2.2. That was to find $f'(0)$, while this is to solve $f'(x) = 0$ for x.)

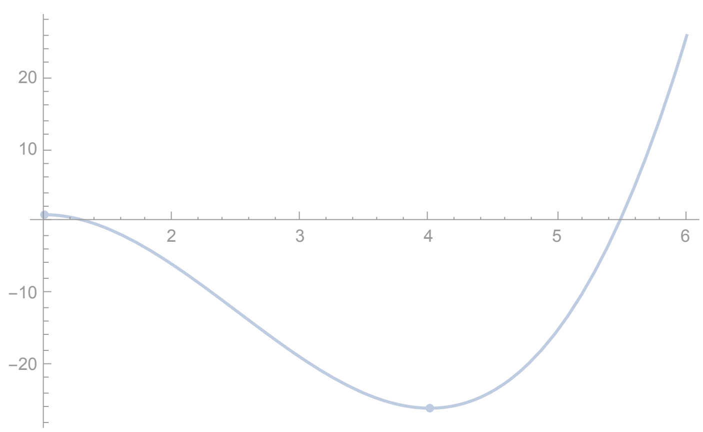
In[733]:=
```
Solve[fp[x] == 0, x]
```

Out[733]= {{x → 1}, {x → 4}}

1.4. Using something like Desmos or a graphing calculator, graph $y = f(x)$ for $-1 \le x \le 6$. Sketch the results below along with the points on the graph that correspond to the x-values you got in 2.3.

```
Show[Plot[f[x], {x, 1, 6}], ListPlot[{{1, f[1]}, {4, f[4]}}, PlotStyle → Larger]]
```

Out[734]=



Out[735]= $-10 + 24x - 15x^2 + 2x^3$

Out[736]= 1

Out[737]= 128

Out[738]= 240

2. Let $f(x) = x^5$ and $g(x) = \frac{1}{x^5}$.

In[739]:=
```
f[x_] := x^5
g[x_] := x^(-5)
```

2.1. What is $f'(x)$ and what is $g'(x)$?

In[741]:=
```
f'[x]
g'[x]
```

Out[741]= $5 x^4$

Out[742]= $-\frac{5}{x^6}$

2.2. What are the fourth derivatives of each, $f^{(4)}(x)$ and $g^{(4)}(x)$?

In[743]:=
```
D[f[x], {x, 4}]
D[g[x], {x, 4}]
```

Out[743]= $120 x$

Out[744]= $\frac{1680}{x^9}$

2.3. What are the sixth derivatives of each, $f^{(6)}(x)$ and $g^{(6)}(x)$?

In[745]:=
```
D[f[x], {x, 6}]
D[g[x], {x, 6}]
```

Out[745]= 0

Out[746]= $\frac{151\,200}{x^{11}}$

3. Let $f(x) = x^{4/3} - 3x^{2/3}$.

In[747]:=
```
f[x_] := x^(4/3) - 3 x^(2/3)
```

3.1. Compute the derivative $f'(x)$.

In[748]:=
```
f'[x]
```

Out[748]= $-\frac{2}{x^{1/3}} + \frac{4 x^{1/3}}{3}$

3.2. For which values of x is the derivative $f'(x)$ defined?
For all real numbers except x=0.

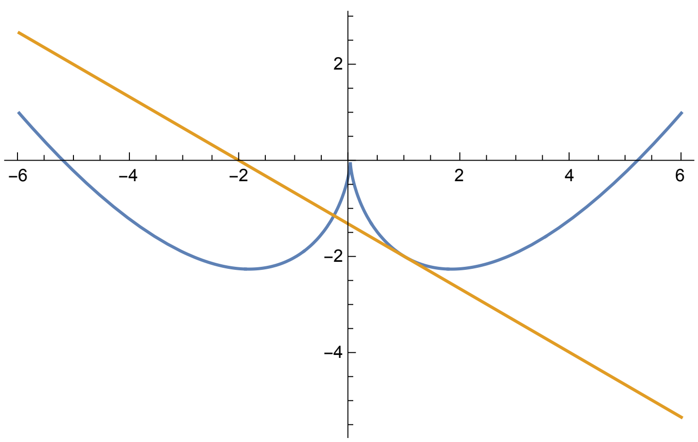
3.3. Find an equation for the tangent line to $y = f(x)$ when $x = 1$.

In[749]:=
```
l[x_] := f[1] + f'[1] (x - 1)
l[x]
```

Out[750]= $-2 - \frac{2}{3}(-1 + x)$

3.4. Using something like Desmos or a graphing calculator, graph $y = f(x)$ for $-6 \le x \le 6$ and this tangent line. Sketch the results below.

In[751]:=
```
Plot[{f[Abs[x]], l[x]}, {x, -6, 6}, PlotStyle → Larger]
```

Out[751]=


3.5. Looking at the graph, what behavior do you see where the derivative is undefined?

It has infinite slope.

4. An airplane's height in miles at time t hours is given by the function $h(t) = 5\sqrt{x} - 3\sqrt[3]{x^2}$.

4.1. Write this function as the difference of two power functions.

In[752]:=
```
h[x_] := 5 √x - 3 ∛(x^2)
h[x_] := 5 x^(1/2) - 3 x^(2/3)
```

4.2. What is the function that represents its instantaneous rate of change of height?

In[754]:=
```
h'[x]
```

Out[754]= $\frac{5}{2\sqrt{x}} - \frac{2}{x^{1/3}}$

4.3. At time $t = 1$ is the plane rising or descending? How fast? In units of mph, the answer is rising, at

In[755]:=
```
N[h'[1]]
```

Out[755]=
```
0.5
```

4.4. At time *t* = 5 is the plane rising or descending? How fast? In units of mph, the answer is:

In[756]:=
```
N[h'[5]]
5 / (2 * Sqrt[5.0]) - 2 / CubeRoot[5.0]
```

Out[756]=
```
-0.0515731065352516
```

Out[757]=
```
-0.0515731065352516
```