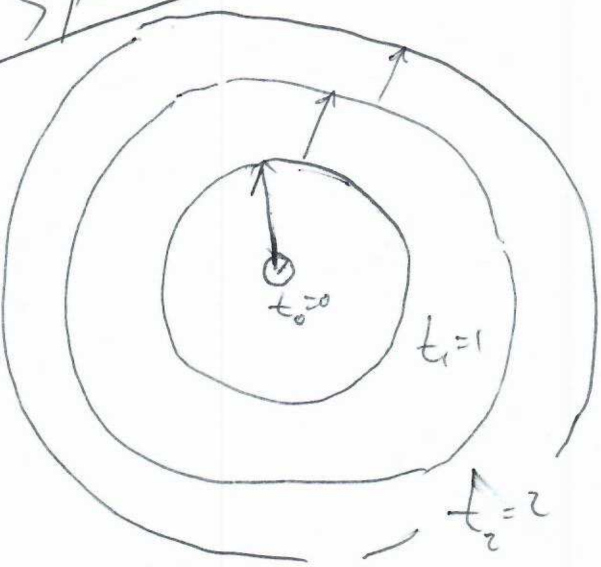
# Sphere




$\frac{dr}{dt} \downarrow$ as time goes on

$$\boxed{\frac{dV}{dt} = 20 \ in^3/s}$$ (given)

Let's relate $\frac{dV}{dt}$ & $\frac{dr}{dt}$; we start by relating V & r.

$$\boxed{V(r) = \tfrac{4}{3}\pi r^3}$$ prior knowledge

$$V(r(t)) = \tfrac{4}{3}\pi\, r(t)^3$$

$$\frac{dV}{dt} = \tfrac{4}{3}\pi \left[(r(t))^3\right]'$$ differentiation with respect to time

$$= \tfrac{4}{\cancel{3}}\pi\, \cancel{3}(r(t))^2 \cdot \frac{dr}{dt}$$

$$\boxed{\frac{dV}{dt} = 4\pi (r(t))^2 \frac{dr}{dt}}$$ related rates

How fast is the radius of the balloon changing when diameter $d = 12$ in ?

$$\boxed{\frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt}}$$

constant

$$= \frac{1}{4\pi r^2} \cdot 20 \ \frac{in^3}{s}$$

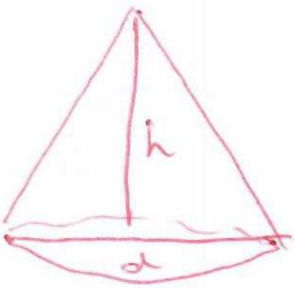
$$= \frac{5}{\pi r^2} \ \frac{in}{s}$$

Now, finally, we substitute in the $d=12$

When $r = \underline{\underline{6}} \left(= \frac{d}{2}\right)$ in ,

$$\frac{dr}{dt} = \frac{5}{\pi 36} \ \frac{in}{s} \approx \boxed{0.044 \ \frac{in}{s}}$$

$$\frac{dV}{dt} = \frac{10\ ft^3}{\text{minute}}$$




$\boxed{h = d}$ (given) $\left( r = \frac{d}{2} = \frac{h}{2} \right)$

How fast is the height of the pile increasing when the pile is 23 ft high?

$\boxed{\text{Asking for } \frac{dh}{dt}}$

We want to relate $\frac{dV}{dt}$ + $\frac{dh}{dt}$;

looking for a relation between $V$ & $h$, & differentiate wrt time $t$.

$$\boxed{V = \tfrac{1}{3}\pi r^2 h}$$

$$V = \tfrac{1}{3}\pi \left(\tfrac{h}{2}\right)^2 \cdot h = \frac{1}{12}\pi h^3$$

$$\therefore \boxed{V(h(t)) = \frac{1}{12}\pi \left(h(t)\right)^3}$$

$$\frac{dV}{dt} = \left[\frac{1}{12}\pi\left(h(t)^3\right)\right]'$$

$$= \frac{\pi}{12}\left[h(t)^3\right]'$$

$$= \frac{\pi}{12} \cdot 3h(t)^2 \cdot \frac{dh}{dt}$$

$$\boxed{\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}}$$ related rates

When $h = 23$ ft,

$$\frac{dh}{dt} = \frac{1}{\frac{\pi}{4}h^2}\frac{dV}{dt} = \frac{1}{\frac{\pi}{4}(23)^2} \cdot 10\ \frac{ft^3}{minute}$$

$$= \frac{40}{\pi(23)^2}\ \frac{ft}{minute}$$

$$= 0.024\ ft/min.$$