Directions: Show our work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answers to each problem (e.g., put a box around them); and clearly separate solutions to each problem from other problems. **Good luck!**

Problem 1: (20 pts) For this problem let

a. (10 pts) **Use the limit definition** to compute the derivative of $f(x) = 2x^2 - 3x + 2$ at $x = 2$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{2(2+h)^2 - 3(2+h) + 2 - 2(2)^2 - 3(2) + 2}{h}$$

2 — 4 — h 4-4=0

$$= \lim_{h\to 0} \frac{(2(4+4h+h^2) - 6 - 3h + 2) - (8 - 6 + 2)}{h} = \lim_{h\to 0} \frac{(\cancel{8} + 8h + 2h^2 \cancel{-6} - 3h \cancel{+2}) \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{5h + 2h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(5+2h)}{\cancel{h}} = \lim_{h\to 0}(5+2h) = \cancel{\lim_{h\to 0}}\ 5 + 2(0)$$

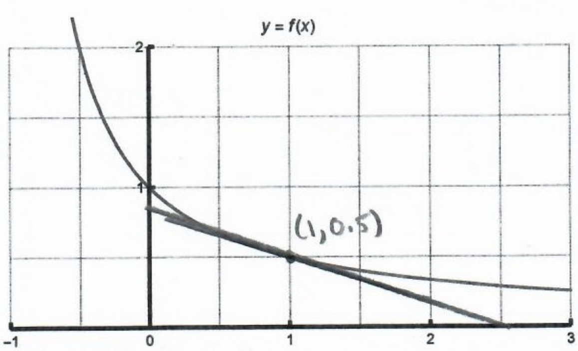
$$= 5 \qquad \boxed{f'(2) = 5}$$

✓

use parentheses

You've passed to the limit

b. (5 pts) Carefully estimate the derivative of $f(x) = \frac{1}{x+1}$ at $x = 1$ using its graph:



The derivative of $f(x) = \frac{1}{x+1}$ at $x=1$ is -1.5

$-2 \cdot (-0.5)$

c. (5 pts) Write the equation of the tangent line at $x = 1$ in point-slope form.

$y - 0.5 = -1.5(x - 1)$ ✓

GYE

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Problem 1: (20 pts) For this problem let

a. (10 pts) **Use the limit definition** to compute the derivative of $f(x) = 2x^2 - 3x + 2$ at $x = 2$.

$h \to 0$

$$\lim_{h\to 2} \frac{(2(2+h)^2 - 3(2+h) + 2) - (2(2)^2 - 3(2) + 2)}{h}$$

$$\lim_{h\to 2} \frac{(2(2+h)^2 - 6 - 3h + 2) - 2(4-3+1)}{h}$$

$$\lim_{h\to 2} \frac{(2(2+h)^2 - 4 - 3h - 2(2)}{h}$$

$$\lim_{h\to 2} \frac{2(4+4h+h^2) - 8 - 3h}{h}$$

$$\lim_{h\to 2} \frac{8+8h+2h^2-8-3h}{h}$$

$$\lim_{h\to 2} \frac{5h+2h^2}{h}$$

$$\lim_{h\to 2} \frac{h(5+2h)}{h}$$

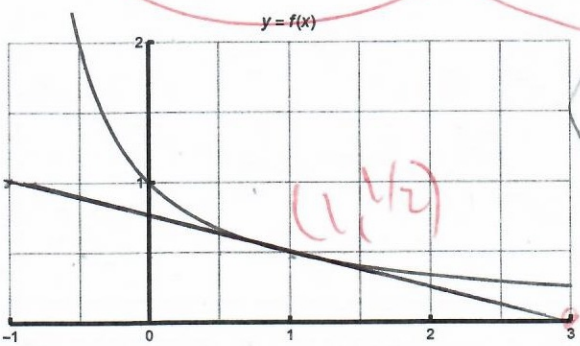
$$\lim_{h\to 2} 5+2h$$

$$5+2(2)=9$$

$\boxed{5}$

-1.5

b. (5 pts) Carefully estimate the derivative of $f(x) = \frac{1}{x+1}$ at $x = 1$ using its graph:



$\approx -\frac{1}{3}$

how?

$-.5$

$(3,0)$ $\longrightarrow$ $m = -\frac{1}{4}$

c. (5 pts) Write the equation of the tangent line at $x = 1$ in point-slope form.

~~$(y - .5) = -\frac{1}{3}(x-1)$~~ $(y - .5) = -\frac{1}{3}(x-1)$

Problem 1: (20 pts) For this problem let

a. (10 pts) **Use the limit definition** to compute the derivative of $f(x) = 2x^2 - 3x + 2$ at $x = 2$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$(2+h)(2+h)$

$4+2h+2h+h^2$

$$\lim_{h\to 0} \frac{2(2+h)^2-3(x+h)+2-4}{h}$$

$$\boxed{\lim_{h\to 0}(5+2h)}$$

$$\lim_{h\to 0} \frac{2(4+4h+h^2)-3x-3h-2}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(5+2h)}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{8+8h+2h^2-3\overset{-6}{x}-3h-2}{h}$$

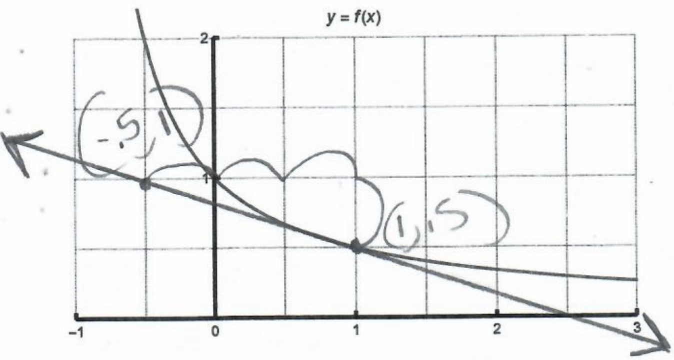
,5

pass to the limit

$= \boxed{5}$

$$\lim_{h\to 0} \frac{2+5h+2h^2-2}{h} \rightarrow \lim_{h\to 0} \frac{5h+2h^2}{h}$$

b. (5 pts) Carefully estimate the derivative of $f(x) = \frac{1}{x+1}$ at $x = 1$ using its graph:



I estimate a derivative of $-\frac{.5}{1.5} = -\frac{1}{3}$

$-.5$

* $(1, \frac{1}{2})$ is not on the line you gave!

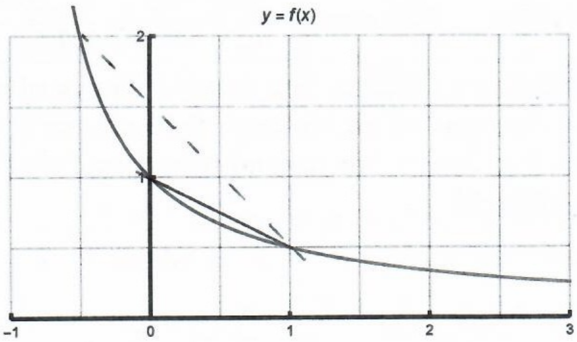
c. (5 pts) Write the equation of the tangent line at $x = 1$ in point-slope form.

$$y - 1 = \left(\frac{-.5}{1.5}\right)(x + .5)$$

$$y - 1 = \left(-\frac{1}{3}\right)(x-(-0.5))$$

Usually the point chosen is the point of tangency: $(1, \frac{1}{2})$ *

Problem 2: (20 pts) Using the same graph of the function $f(x) = \frac{1}{x+1}$



Nice work

a. (4 pts) compute the average rate of change over the period from 0 to 1.

$$\frac{f(1)-f(0)}{1-0} \qquad \frac{\frac{1}{1+1}-\frac{1}{0+1}}{1} \qquad \frac{.5-1}{1} \qquad -0.5$$

b. (4 pts) How does the **average** rate of change compare to the **instantaneous** rate of change

i. at $x = 0$?

The instantanious rate would be a lot steeper it would probably be around x-1 wheras the average is -0.5

ii. at $x = 1$?

The average would be steeper at -0.5 whereas the instantanious would be around x $-\frac{1}{3}$

c. (4 pts) Find an interval on which the average rate of change is -1.

-.5 to 1

good, but point out your method (above, dashed line)

d. (4 pts) Write the equation of the secant line joining points at $x = 0$ and $x = 1$ in point-point form.

$$y-.5=\frac{.5-1}{1-0}(x-1)$$

(0, 1)
(1, .5)

parentheses are important!

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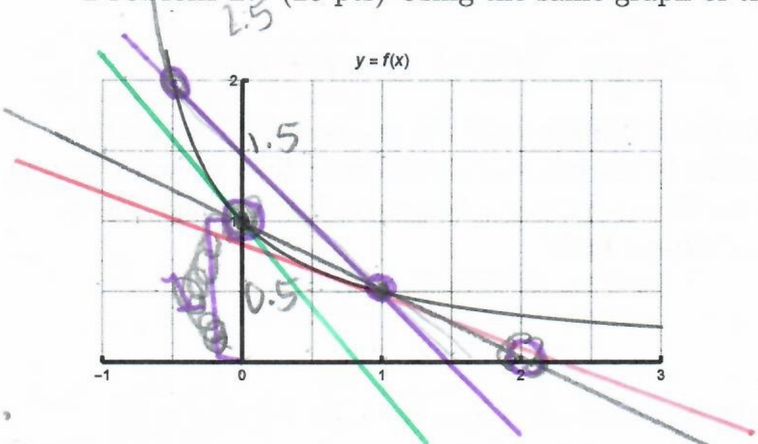
Problem 2: (20 pts) Using the same graph of the function $f(x) = \frac{1}{x+1}$



X	y	
0	1	(1) (a)
1	0.5	(2) (b)
2	0	

a. (4 pts) compute the average rate of change over the period from $\boxed{0 \text{ to } 1.}$

$AV_{[a,b]} = \frac{S(b)-S(a)}{b-a}$ $\quad AV_{[0,1]} = \frac{0.5-1}{1-0} = \frac{-0.5}{1} = -0.5$

$\boxed{AV_{[0,1]} = -0.5}$ ✓

b. (4 pts) How does the **average** rate of change compare to the **instantaneous** rate of change

i. at $x = 0$?

Instantaneous rate of change is the slope of the tangent line. At x=0 the slope of the tangent line is in the same direction (negative) but is steeper than the average slope (secant line) of the two points. Slope of instantaneous closer to -1. ✓

well done

ii. at $x = 1$?

The slope of the tangent line at x=1 is flatter than the average & in the same diretion (negative). However, it is closer to $-\frac{1}{3}$ than -1. ✓

c. (4 pts) Find an interval on which the average rate of change is -1.

X	y	
-0.5	2	(1)
1	0.5	(2)

$\frac{y-y_1}{x-x_1} = \frac{0.5-2}{1-(-0.5)} = \frac{-1.5}{1.5} = -1$

$AV_{[-0.5,1]} = -1$

great!

d. (4 pts) Write the equation of the secant line joining points at $x = 0$ and $x = 1$ in point-point form.

$y - y_1 = \frac{y-y_1}{x-x_1}(x-x_1)$

$\boxed{\cancel{0.5} \; y - 1 = \frac{0.5-1}{1-0}(\cancel{x}-0)}$

X	y	
0	1	(1)
1	0.5	(2)

-1

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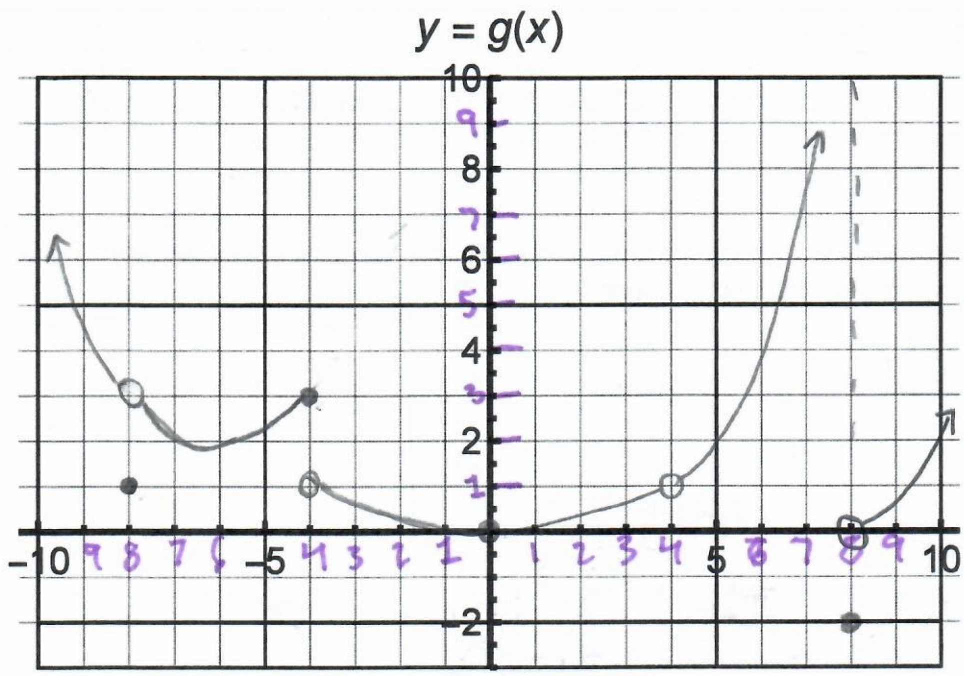
Problem 3: (10 pts) Draw the graph of a function g consistent with the following:



X	y
-8	1
-4	3
0	0
4	DNE
8	-2

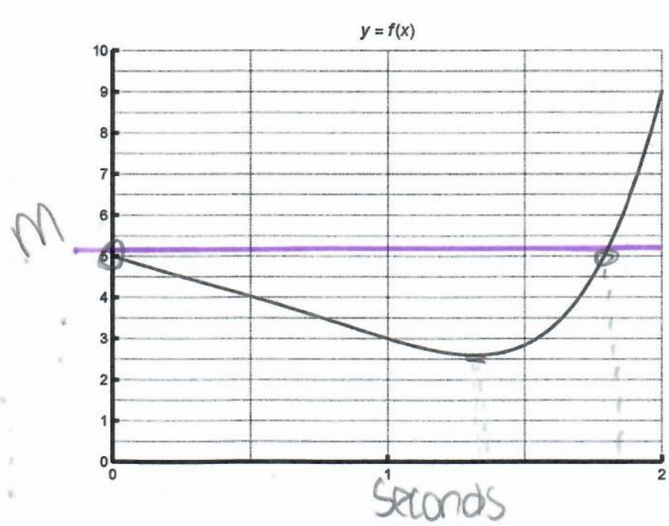
$g(-8) = 1$
$g(-4) = 3$
$g(0) = 0$
$g(4) = DNE$
$g(8) = -2$

$\lim_{x\to-8} g(x) = 3$
$\lim_{x\to-4^-} g(x) = 3$
$\lim_{x\to-4^+} g(x) = 1$
$\lim_{x\to0} g(x) = 0$
$\lim_{x\to4} g(x) = 1$
$\lim_{x\to8^-} g(x) = DNE$
$\lim_{x\to8^+} g(x) = 0$

Would have checked these with more time.

you didn't need it! :)

Problem 4: (10 pts) The height of a bird is given by this graph, with time on the x-axis, in seconds(s), and height(m) on the y-axis. To the right of the graph, describe how its height changes over time (4pts).



Within two seconds a bird starts at five meters in the air and decends to 2.5 meters at 1.4 seconds & then flies upwards to 9 meters for the rest of the two total seconds captured.

Talk about rates

a. (3 pts) At what time is its instantaneous rate of change of height greatest?

two seconds because steepest positive slope of tangent line.

b. (3 pts) Estimate the time t at which the average rate of change of height is 0 over interval $[0, t]$.

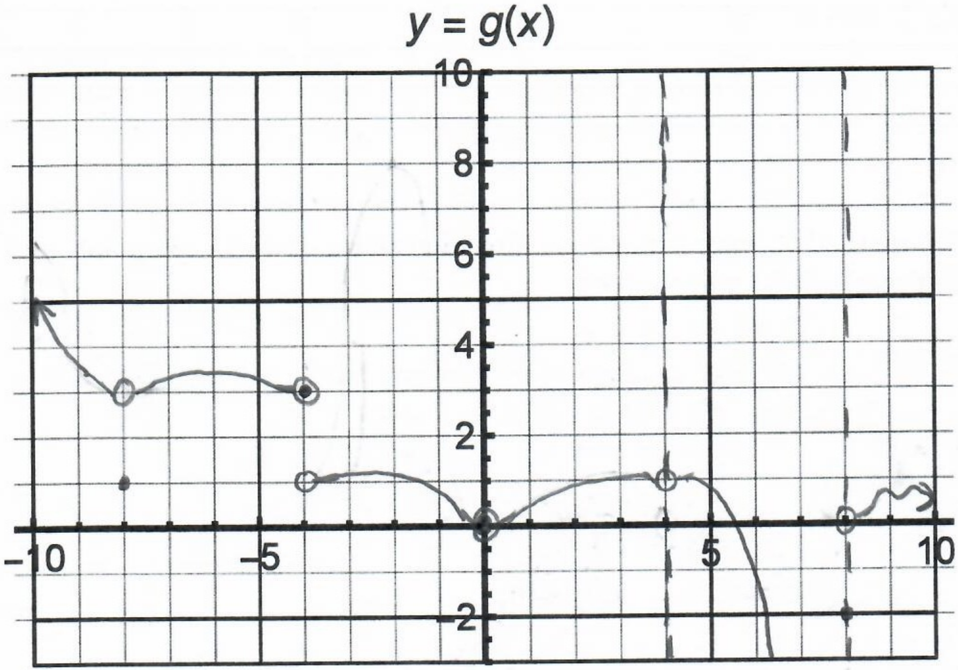
19.5

x	y
0	5
1.8	5

$AV_{[0,1.8]} = 0$ flat slope = zero

good

Problem 3: (10 pts) Draw the graph of a function g consistent with the following:



$g(-8) = 1$ ✓
$g(-4) = 3$ ✓
$g(0) = 0$ ✓
$g(4) = DNE$ ✓
$g(8) = -2$ ✓

$\lim_{x\to -8} g(x) = 3$ ✓
$\lim_{x\to -4^-} g(x) = 3$ ✓
$\lim_{x\to -4^+} g(x) = 1$ ✓
$\lim_{x\to 0} g(x) = 0$ ✓
$\lim_{x\to 4} g(x) = 1$ ✓
$\lim_{x\to 8^-} g(x) = DNE$ ✓
$\lim_{x\to 8^+} g(x) = 0$ ✓

Good

Problem 5: (20 pts) Water drains from a tank over a period of minutes, as shown in the data below:

X	time t (minutes)	0	1	2	3	4
Y	depth $d(t)$ (meters)	4	2	1	0.5	0.25
slope	$\approx d'(t)$	-4	-2	-1	-0.5	-0.25

a. (12 pts) Choose a reasonable method to approximate the first derivative of d at each point, and so fill in the table above. (What method did you choose?)

$f'(x)$

$\lim_{h\to 0} f(x) = \frac{f(x+h)-f(x)}{h}$

X	y
0	4

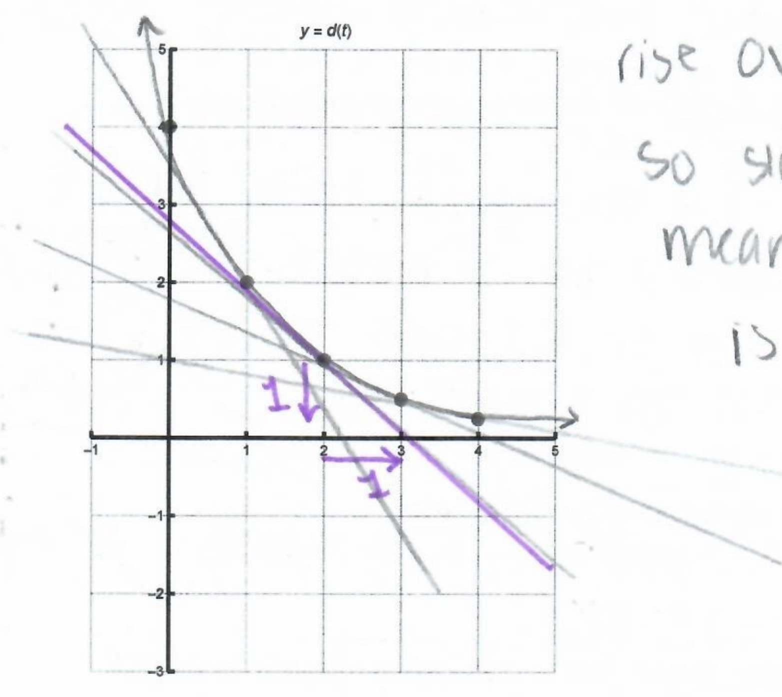
Method: Drew tangent lines on provided graph + did rise/run.

good!

b. (4 pts) What is the average rate of change over the entire interval (include the units)?

$AV_{[0,4]} = \frac{f(4)-f(0)}{4-0} = \frac{0.25-4}{4-0} = -0.9375 \frac{m}{min}$ ✓

c. (4 pts) **Explain** your derivative estimate at $t = 2$ from the table, using the graph of the data below:



rise over run is $\frac{-1}{1}$

so slope of tangent is -1

meaning derivative @ t=2 is $\boxed{-1}$

Well done