

MAT128: Exam 1 (Spring 2024): Sections 1.1-1.3

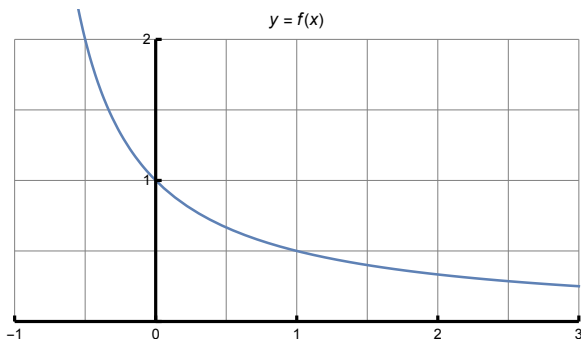
Name:

Directions: Show our work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answers to each problem (e.g., put a box around them); and clearly separate solutions to each problem from other problems. **Good luck!**

Problem 1: (20 pts)

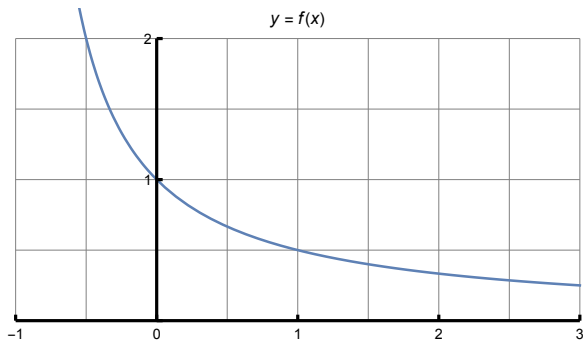
a. (10 pts) Use **the limit definition** to compute the derivative of $f(x) = 2x^2 - 3x + 2$ at $x = 2$.

b. (5 pts) Carefully estimate the derivative of $f(x) = \frac{1}{x+1}$ at $x = 1$ using its graph:



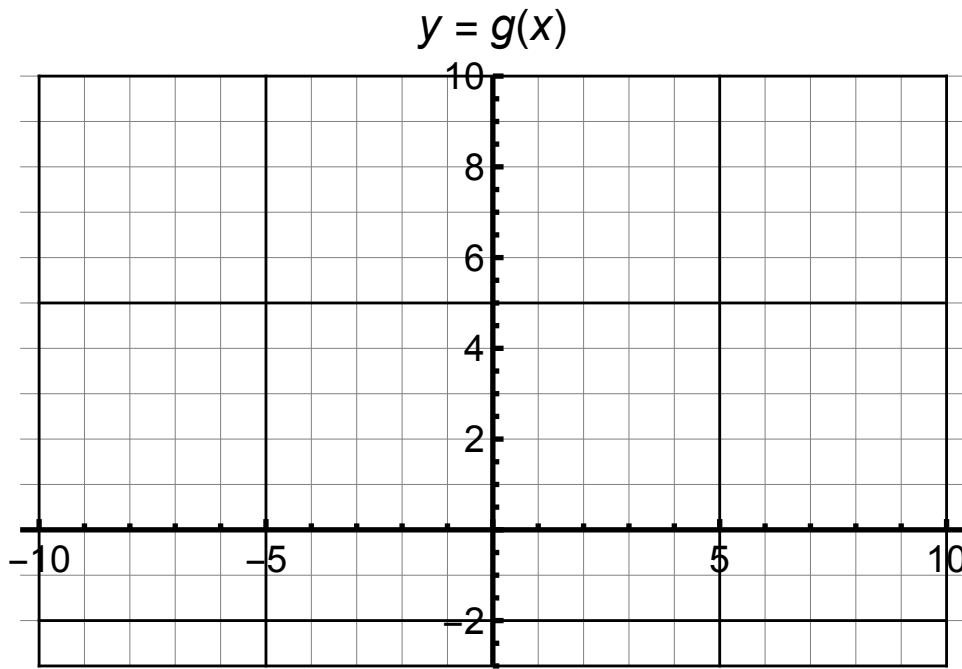
c. (5 pts) Write the equation of the tangent line at $x = 1$ in point-slope form.

Problem 2: (20 pts) Using the same graph of the function $f(x) = \frac{1}{x+1}$



- a. (5 pts) compute the average rate of change over the period from 0 to 1.
- b. (5 pts) How does the **average** rate of change compare to the **instantaneous** rate of change
- at $x = 0$?
 - at $x = 1$?
- c. (5 pts) Find an interval on which the average rate of change is -1.
- d. (5 pts) Write the equation of the secant line joining points at $x = 0$ and $x = 1$ in point-point form.

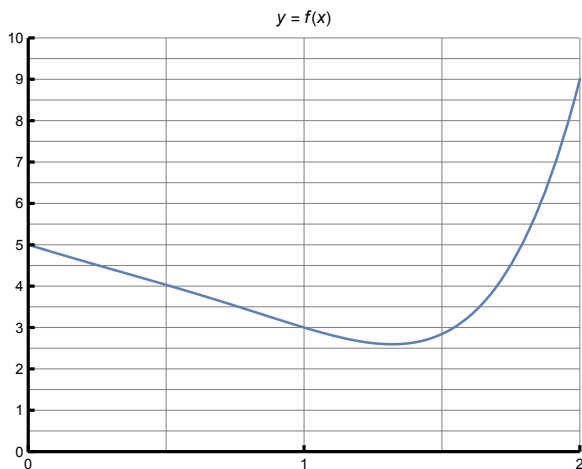
Problem 3: (10 pts) Draw the graph of a function g consistent with the following:



$$\begin{aligned} g(-8) &= 1 \\ g(-4) &= 3 \\ g(0) &= 0 \\ g(4) &= DNE \\ g(8) &= -2 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -8} g(x) &= 3 \\ \lim_{x \rightarrow -4^-} g(x) &= 3 \\ \lim_{x \rightarrow -4^+} g(x) &= 1 \\ \lim_{x \rightarrow 0} g(x) &= 0 \\ \lim_{x \rightarrow 4} g(x) &= 1 \\ \lim_{x \rightarrow 8^-} g(x) &= DNE \\ \lim_{x \rightarrow 8^+} g(x) &= 0 \end{aligned}$$

Problem 4: (10 pts) The height of a bird is given by this graph, with time on the x -axis, in seconds(s), and height(m) on the y -axis. To the right of the graph, describe how its height changes over time (4pts).



- a. (3 pts) At what time is its instantaneous rate of change of height greatest?

- b. (3 pts) Estimate the time t at which the average rate of change of height is 0 over interval $[0, t]$.

Problem 5: (20 pts) Water drains from a tank over a period of minutes, as shown in the data below:

time t (minutes)	0	1	2	3	4
depth $d(t)$ (meters)	4	2	1	0.5	0.25
$\approx d'(t)$					

a. (12 pts) Choose a reasonable method to approximate the first derivative of d at each point, and so fill in the table above. (What method did you choose?)

b. (4 pts) What is the average rate of change over the entire interval (include the units)?

c. (4 pts) **Explain** your derivative estimate at $t = 2$ from the table, using the graph of the data below:

