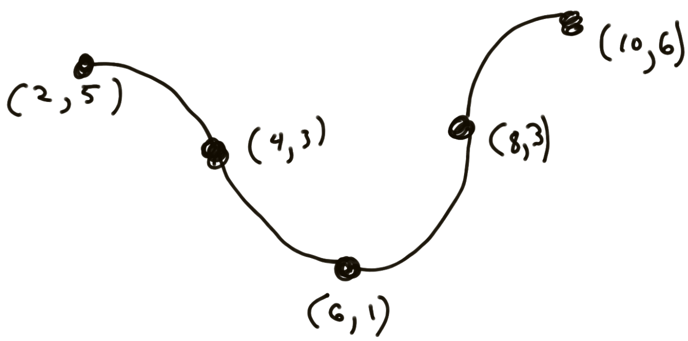
Section 1.6 – Second Order Derivatives

1. Given the following graph of $y = f(x)$, fill in the following table.

	f	f'	f''
Values (intervals) where negative	N/A	$(2,6)$	$(2,4) \cup (8,10)$
Values where equal to zero	N/A	$\{2,6,10\}$	$\{4,8\}$
Values (intervals) where positive	$(2,10)$	$(6,10)$	$(4,8)$

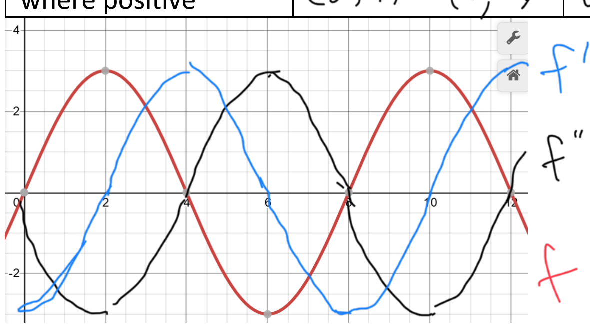


2. If f is represented by the following table, fill in the missing rows on the table with the best approximation: forward ‖ centered ‖ backward

x	0	2	4	6
$f(x)$	4	6	7	6
$f'(x)$	1	$3/4$	0	$-1/2$
$f''(x)$	$-3/8$	$-1/4$	$-5/16$	$-1/4$

3. Given the following graph of $y = f(x)$, fill in the following table and draw graphs of f' and f''.

	f	f'	f''
Values (intervals) where negative	$(4,8)$	$(2,6)$ ~~or~~ $(10,12)$	$(0,4) \cup (8,12)$
Values where equal to zero	$\{0,4,8,12\}$	$\{2,6,10\}$	$\{0,4,8,12\}$
Values (intervals) where positive	$(0,4) \cup (8,12)$	$(0,2) \cup (6,10)$	$(4,8)$



$$f''(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h} = \lim_{h\to 0}\frac{3(\cancel{x^2}+2xh+h^2)-\cancel{3x^2}}{h}$$

$$= \lim_{h\to 0}\frac{6xh+3h^2}{h} = \lim_{h\to 0}\frac{\cancel{h}(6x+3h)}{\cancel{h}} = \lim_{h\to 0}(6x+3h) = \boxed{6x}$$

4. Given that $y = f(x) = x^3$, use the algebraic definition to find a formula for $y = f'(x)$ and $y = f''(x)$. *Show all of your work!*

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$

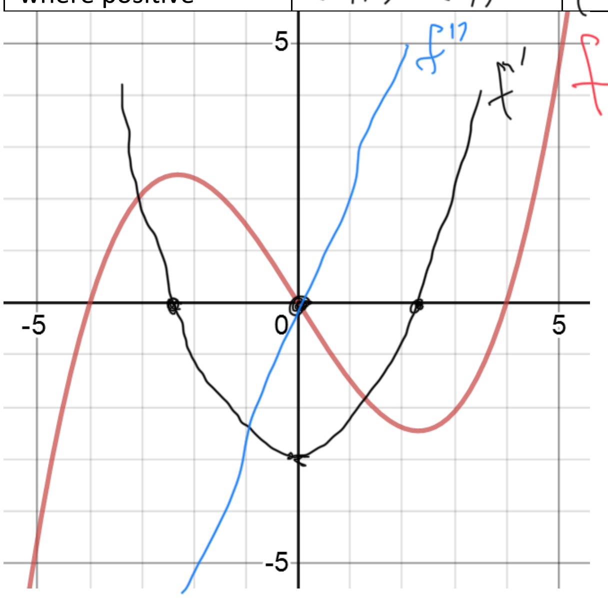
$$= \lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-x^3}}{h} = \lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

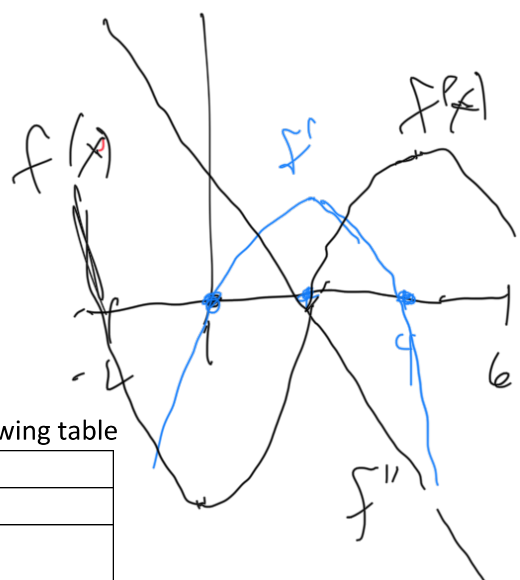
$$= \lim_{h\to 0}(3x^2+3xh+h^2) = \boxed{3x^2}$$

f'' in blue

5. Given the following graph of $y = f(x)$, fill in the following table and draw a graph of f' and f''.

	f	f'	f''
Values (intervals) where negative	$(-\infty,-4)\cup(0,4)$	$(-2.5,2.5)$	$(-\infty,0)$
Values where equal to zero	$\{-4,0,4\}$	$\{-2.5,2.5\}$	$\{0\}$
Values (intervals) where positive	$(-4,0)\cup(4,\infty)$	$(-\infty,-2.5)\cup(2.5,\infty)$	$(0,\infty)$





6. Optional: Draw a graph of f that is consistent with the following table

	f'	f''
Values (intervals) where negative	(-2,0), (4,6)	(2,6)
Values where equal to zero	x = 0, x=4	X =2
Values (intervals) where positive	(0,4)	(-2,2)

Might smooth out those graphs a little.... :)