Homework 2.2

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Section 2.2, pp. 57--, #1, 2a-f, 10, 12, 13

Some problems are solved in Octave:

https://octav.onl/youat248

#2

2. (i) Decide whether or not the hypotheses of the Mean

Value Theorem are met for the function over the interval. (ii) If the hypotheses are met, find a value c as guaranteed by the theorem.

(a) $f(x) = 3 - x - \sin x$; $[2, 3]$

(b) $g(x) = 3x^4 - 2x^3 - 3x + 2$; $[0, 1]$

(c) $g(x) = 3x^4 - 2x^3 - 3x + 2$; $[0, 0.9]$ [𝕊]

(d) $h(x) = 10 - \cosh(x)$; $[-3, -2]$ [𝔸]

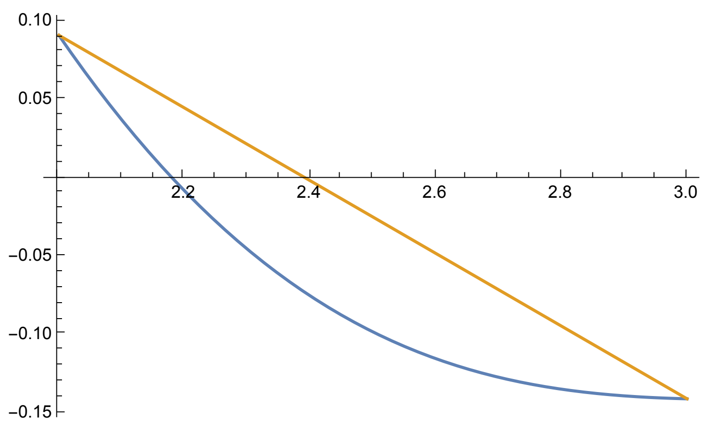
(e) $f(t) = \sqrt{4 + 5\sin t} - 2.5$; $[-6, -5]$

(f) $g(t) = \frac{3t^2 \tan t}{1-t^2}$; $[20, 23]$ [𝕊]

In[∘]:=
```
meanValueTheoremTesterFinder[f_, a_, b_] :=
 Module[{fa = f[a], fb = f[b], m},
  m = (fb - fa) / (b - a);
  p1 = Plot[{f[x], m * (x - a) + fa}, {x, a, b}];
  Print[N[Solve[f'[x] == m, x]]];
  p1
 ]
f[x_] := 3 - x - Sin[x]
meanValueTheoremTesterFinder[f, 2, 3]
f[x_] := 3 x^4 - 2 x^3 - 3 x + 2
meanValueTheoremTesterFinder[f, 0, 1]
f[x_] := 3 x^4 - 2 x^3 - 3 x + 2
meanValueTheoremTesterFinder[f, 0, 0.9]
f[x_] := 10 - Cosh[x]
meanValueTheoremTesterFinder[f, -3, -2]
f[x_] := Sqrt[4 + 5 Sin[x]] - 2.5
meanValueTheoremTesterFinder[f, -6, -5]
f[x_] := (3 x^2 Tan[x]) / (1 - x^2)
meanValueTheoremTesterFinder[f, 20, 23]
```

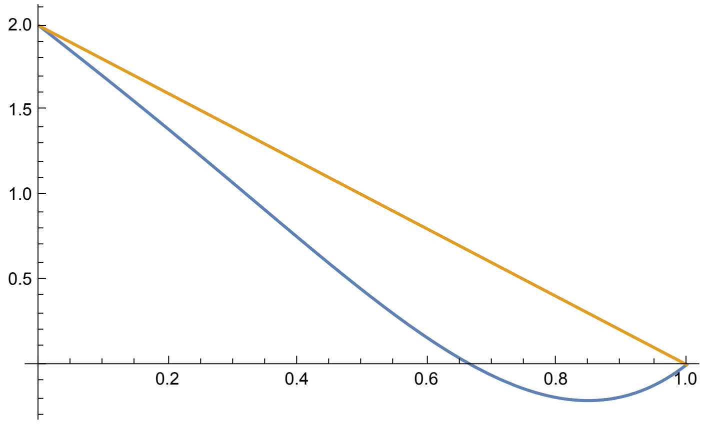
$\{\{x \to -2.44679 + 6.28319\, c_1 \text{ if } c_1 \in \mathbb{Z}\}, \{x \to 2.44679 + 6.28319\, c_1 \text{ if } c_1 \in \mathbb{Z}\}\}$

Out[∘]=



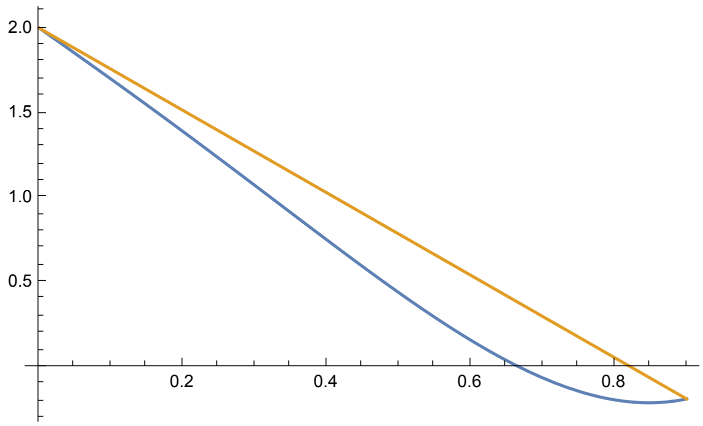
$\{\{x \to 0.680143\}, \{x \to -0.0900716 - 0.338246\, i\}, \{x \to -0.0900716 + 0.338246\, i\}\}$

Out[•]=



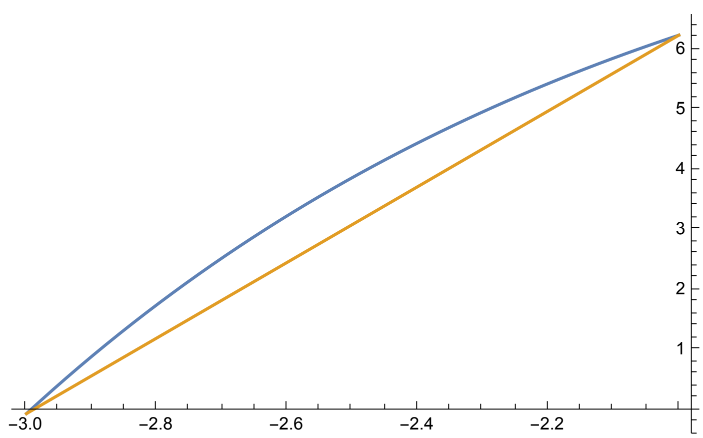
{{x → -0.0610465 - 0.26875 i}, {x → -0.0610465 + 0.26875 i}, {x → 0.622093}}

Out[•]=



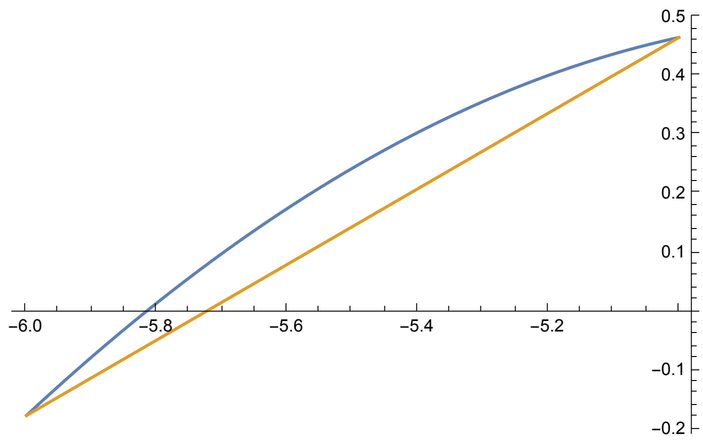
{{x → (2.54079 + 3.14159 i) + (0. + 6.28319 i) c_1 if $c_1 \in \mathbb{Z}$},
{x → -2.54079 + (0. + 6.28319 i) c_1 if $c_1 \in \mathbb{Z}$}}

Out[•]=



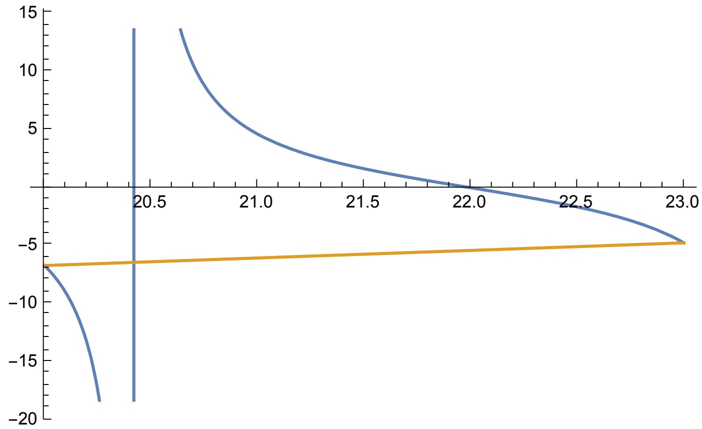
{{x → -1.5708 + 0.277136 i}, {x → 0.787363}}

Out[∘]=



$$\text{Solve}\left[\frac{3.\ x^2\ \text{Sec}[x]^2}{1.-1.\ x^2}+\frac{6.\ x^3\ \text{Tan}[x]}{\left(1.-1.\ x^2\right)^2}+\frac{6.\ x\ \text{Tan}[x]}{1.-1.\ x^2}==0.651607,\ x\right]$$

Out[∘]=



#10

10. Let $f(x) = \frac{3x^2-1}{6x+4}$. [S]

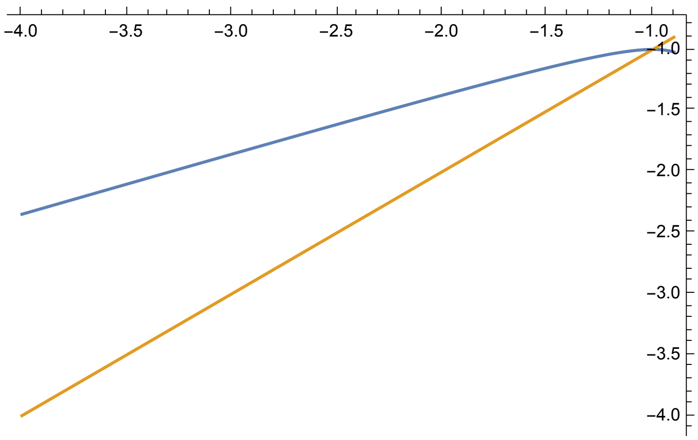
(a) Show that f has a unique fixed point on $[-4, -0.9]$.

(b) Use fixed point iteration to find an approximation to the fixed point that is accurate to within 10^{-2}.

In[∘]:=
```
fixedPointFinder[f_, a_, b_] :=
 Module[{fa = f[a], fb = f[b], m},
  m = (fb - fa) / (b - a);
  p1 = Plot[{f[x], x}, {x, a, b}];
  Print[N[Solve[f'[x] == m, x]]];
  p1
 ]
f[x_] := (3 x^2 - 1) / (6 x + 4)
fixedPointFinder[f, -4, -0.9]
```

```
{{x → -1.54858}, {x → 0.21525}}
```

Out[∘]=



In[∘]:=
```
results = fixedPoint[f, -1.5, maxits → 1000, verbose → True, tolerance → 10^-2]
```

```
Root at c.

-1.5
```

a_k	c_k	f[c_k]	slope
-1.5	-1.15	-1.023275862068966	0.3620689655172413
-1.15	-1.023275862068966	-1.00075960710217	0.1776792908944438
-1.023275862068966	-1.00075960710217	-1.000000863536579	0.03369759166031787
-1.00075960710217	-1.000000863536579	-1.000000000001118	0.001138112400361676

```
  c   = -1.000000863536579

 Δc   = ±0.000758744

f[c]  = -1.000000000001118

f'[c] = 0.001138112400361676
```

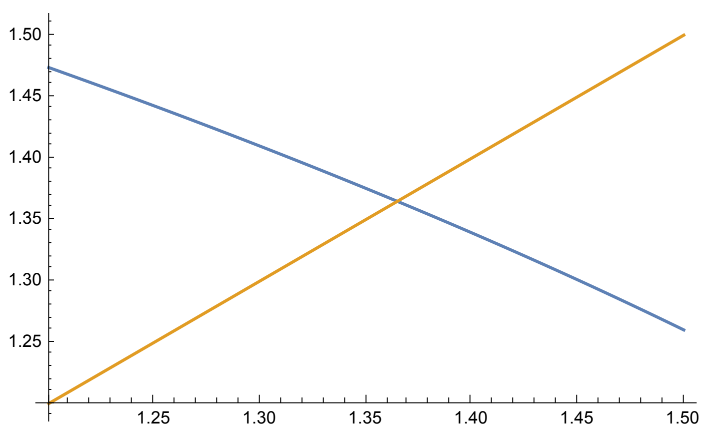
Out[•]=
```
{{-1.5, -1.15, -1.02328, -1.00076}, {-1.15, -1.02328, -1.00076, -1.},
 {-1.02328, -1.00076, -1., -1.}, {0.362069, 0.177679, 0.0336976, 0.00113811}}
```

#12

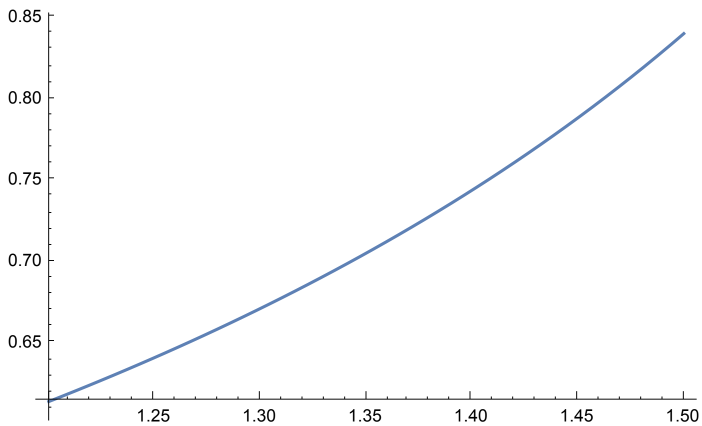
12. Show that the fixed point iteration method applied to $f(x) = \sqrt[3]{8-4x}$ will converge to a root of $g(x) = x^3 + 4x - 8$ for any initial value $x_0 \in [1.2, 1.5]$. [S]

In[•]:=
```
f[x_] := CubeRoot[8 - 4 x]
fixedPointFinder[f, 1.2, 1.5]
Plot[Abs[f'[x]], {x, 1.2, 1.5}]
results = fixedPoint[f, 1.2, maxits → 1000, verbose → True, tolerance → 10^-2];
results = fixedPoint[f, 1.5, maxits → 1000, verbose → True, tolerance → 10^-2];
```

```
{{x → 1.35975}, {x → 2.64025}}
```

Out[•]=


Out[•]=



```
Root at c.

1.2
```

a_k	c_k	$f[c_k]$	slope
1.2	1.473612599456155	1.281706221158079	-0.7013799023857757
1.473612599456155	1.281706221158079	1.421632572384772	0.7291386167965439
1.281706221158079	1.421632572384772	1.322577933919723	-0.7079055345663305
1.421632572384772	1.322577933919723	1.394140178228879	0.7224522285688416
1.322577933919723	1.394140178228879	1.343210407107426	-0.7116849340475126
1.394140178228879	1.343210407107426	1.379840066106201	0.7192189988724702
1.343210407107426	1.379840066106201	1.353696269287307	-0.7137330112673972
1.379840066106201	1.353696269287307	1.372457436507753	0.7176144823343934
1.353696269287307	1.372457436507753	1.359046739351805	-0.7148114506080857
1.372457436507753	1.359046739351805	1.368659609067846	0.7168061141234072
1.359046739351805	1.368659609067846	1.361782834088909	-0.7153717029434357

```
  c   = 1.368659609067846

 Δc   = ±0.00961287

f[c]  = 1.361782834088909

f'[c]  = -0.7153717029434357

Root at c.

1.5
```

a_k	c_k	$f[c_k]$	slope
1.5	1.259921049894873	1.435861930498375	0.732845926419037
1.259921049894873	1.435861930498375	1.311641451496065	-0.7060353374168441
1.435861930498375	1.311641451496065	1.401602619500796	0.7242056118867339
1.311641451496065	1.401602619500796	1.337672785805802	-0.7106381021156994
1.401602619500796	1.337672785805802	1.383707188404067	0.7200769959435988
1.337672785805802	1.383707188404067	1.350876659014053	-0.7131737904045673
1.383707188404067	1.350876659014053	1.374450400990172	0.7180433095084097
1.350876659014053	1.374450400990172	1.357606516073488	-0.7145189310100635
1.374450400990172	1.357606516073488	1.369683969177791	0.7170230124488725
1.357606516073488	1.369683969177791	1.361045930425527	-0.715220227117788
1.369683969177791	1.361045930425527	1.367235136066841	0.71650588968381

```
  c   = 1.361045930425527

 Δc   = ±0.00863804

f[c]  = 1.367235136066841

f'[c]  = 0.71650588968381
```

#13

13. Show that fixed point iteration is guaranteed to converge to the fixed point of

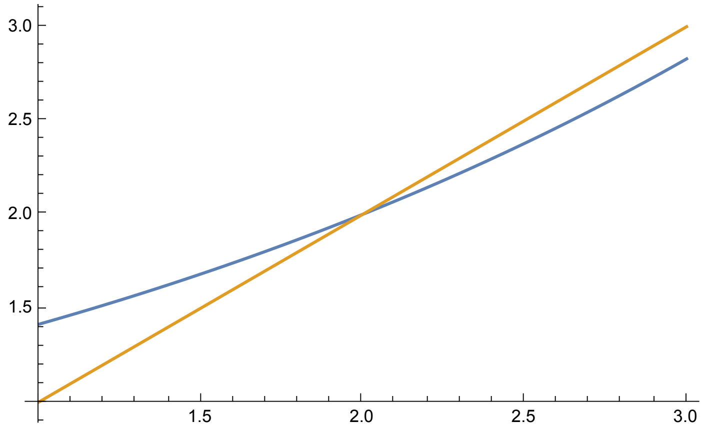
$$f(x) = (\sqrt{2})^x$$

for any $x_0 \in [1,3]$. HINT: $f'(x) = \frac{1}{2}\ln(2)\cdot(\sqrt{2})^x$.

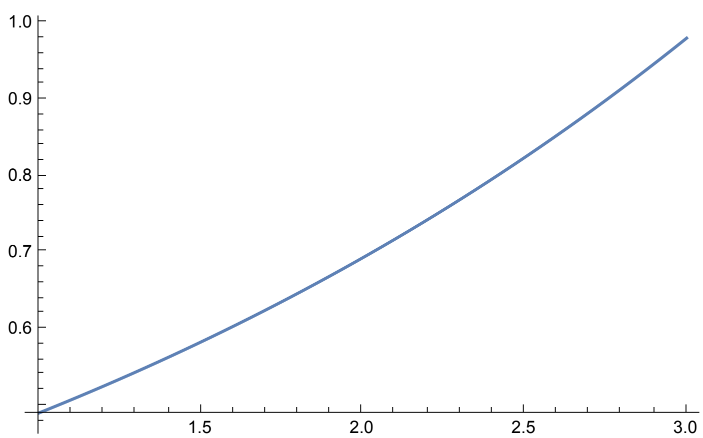
In[∘]:=
```
f[x_] := 2^(x/2)
fixedPointFinder[f, 1, 3]
Plot[Abs[f'[x]], {x, 1, 3}]
results = fixedPoint[f, 1.0, maxits → 1000, verbose → True, tolerance → 10^-2];
results = fixedPoint[f, 3.0, maxits → 1000, verbose → True, tolerance → 10^-2];
```

$\{\{x \to 2.\ (1.02877 + (0. + 9.06472\ i)\ c_1)$ if $c_1 \in \mathbb{Z}\}\}$

Out[∘]=



Out[∘]=



Root at c.

1.

a_k	c_k	$f[c_k]$	slope
1.	1.414213562373095	1.632526919438153	0.5270550674736624
1.414213562373095	1.632526919438153	1.760839555880028	0.5877452399929787
1.632526919438153	1.760839555880028	1.84091086929101	0.6240329528826577
1.760839555880028	1.84091086929101	1.89271269682851	0.6469461450147154
1.84091086929101	1.89271269682851	1.9269997018471	0.6618879419605245
1.89271269682851	1.9269997018471	1.950034773805817	0.6718309734614564
1.9269997018471	1.950034773805817	1.965664886517319	0.678535441066265
1.950034773805817	1.965664886517319	1.976341754409702	0.6830960268461257
1.965664886517319	1.976341754409702	1.983668399303821	0.6862166852645228
1.976341754409702	1.983668399303821	1.988711773413953	0.6883606593490779

c = 1.983668399303821

△c = ±0.00732664

f[c] = 1.988711773413953

f'[c] = 0.6883606593490779

Root at c.

3.

a_k	c_k	$f[c_k]$	slope
3.	2.82842712474619	2.665144142690225	−0.951682961624434
2.82842712474619	2.665144142690225	2.518512814015087	−0.898019663952981
2.665144142690225	2.518512814015087	2.393723319824124	−0.851042511300117
2.518512814015087	2.393723319824124	2.292404542149576	−0.811917528245623
2.393723319824124	2.292404542149576	2.213304997816393	−0.7806997493323809
2.292404542149576	2.213304997816393	2.153453988736905	−0.7566542839663479
2.213304997816393	2.153453988736905	2.109245457346235	−0.7386430416228343
2.153453988736905	2.109245457346235	2.077174944852915	−0.725437183377879
2.109245457346235	2.077174944852915	2.054215407641809	−0.7159080234807233
2.077174944852915	2.054215407641809	2.03793452874122	−0.7091118061697498
2.054215407641809	2.03793452874122	2.026467818007143	−0.704305388185243
2.03793452874122	2.026467818007143	2.018430496092366	−0.7009265430314081
2.026467818007143	2.018430496092366	2.012815933865572	−0.6985613225806829

c = 2.018430496092366

△c = ±0.00803732

f[c] = 2.012815933865572

f'[c] = −0.6985613225806829