

a)

$$b) L'_i(x) = f(x_0) \frac{1}{x_0 - x_1} + f(x_1) \frac{1}{x_1 - x_0} = \frac{f(x_0) - f(x_1)}{x_1 - x_0}$$

$$c) L'_i(x_0 + \frac{1}{2}h) = \frac{f(x_0 + \frac{1}{2}h) - f(x_0)}{\frac{1}{2}h}$$

#5 $x = x_0 + \theta h$ $\theta = 0, 1, \frac{1}{2}$

$$L(x) = f(x_0) \frac{x - x_1}{x_0 - x_1} + f(x_1) \frac{x - x_0}{x_1 - x_0}$$

$$= f(x_0) \frac{(x_0 + \theta h) - (x_0 + h)}{-h} + f(x_1) \frac{(x_0 + \theta h) - x_0}{(x_0 + h) - x_0}$$

$$L(x(\theta)) = f(x_0)(-\theta + 1) + f(x_1)(\theta)$$

$$\frac{dL}{dx} = \frac{dL}{d\theta} \cdot \frac{d\theta}{dx}$$

$$= (f(x_1) - f(x_0)) \cdot \frac{1}{h}$$

$$= \frac{f(x_0 + h) - f(x_0)}{h}$$

$x = x_0 + \theta h$
 $\frac{d}{dx}(x_0 + \theta h) = \frac{d\theta}{dx} \cdot h$
 $1 = \frac{d\theta}{dx} \cdot h$
 $\frac{d\theta}{dx} = \frac{1}{h}$

#6

x_0	y_0	$\frac{y_1 - y_0}{h}$	$\frac{\frac{y_1 - y_0}{h} - \frac{y_1 - y_0}{h}}{2h}$
x_1	y_1	$\frac{y_2 - y_1}{h}$	
x_2	y_2		

HHE
Horrible Huge Expression

$$N_2(x) = y_0 + \frac{y_1 - y_0}{h}(x - x_0) + HHE(x - x_0)(x - x_1)$$

$$N_2'(x) = \frac{y_1 - y_0}{h} + HHE(2x - x_0 - x_1)$$

$$N_2'(x_0 + \frac{1}{2}h) = \frac{y_1 - y_0}{h} + \frac{2(x_0 + \frac{1}{2}h) - x_0 - (x_0 + h)}{2h} \cdot HHE$$

$$= \frac{y_1 - y_0}{h} + 0 \cdot HHE = \frac{f(x_1) - f(x_0)}{h}$$

#7

$$L_2(\theta) = \frac{(\theta - 1)(\theta - 2)}{2} f(x_0) + \frac{\theta(\theta - 2)}{-1} f(x_1) + \frac{\theta(\theta - 1)}{2} f(x_2)$$

$x = x_0 + \theta h$
 $x - \theta h = x_0$
 $\theta = 0, \theta = 1, \theta = 2$
 $\frac{d\theta}{dx} = \frac{1}{h}$

$$\frac{dL}{dx} = \frac{dL}{d\theta} \cdot \frac{d\theta}{dx} = \frac{1}{h} \left[\left(\frac{2\theta - 3}{2} \right) y_0 + (2 - 2\theta) y_1 + \left(\frac{2\theta - 1}{2} \right) y_2 \right]$$

$$\frac{d}{d\theta} \left(\frac{dL}{dx} \right) = \frac{d}{d\theta} \left(\frac{1}{h} \left[\left(\frac{2\theta - 3}{2} \right) y_0 + (2 - 2\theta) y_1 + \left(\frac{2\theta - 1}{2} \right) y_2 \right] \right)$$

$$= \frac{1}{h^2} \left[y_0 - 2y_1 + y_2 \right]$$

$$= \frac{f(x_0) - 2f(x_1) + f(x_2)}{h^2}$$