

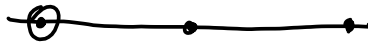
Deriving error terms for first derivative
three point methods.

centered:



$$f'(x_0) - p_2'(x_0) = \frac{f^{(3)}(\xi_c)}{3!} (-1)(1)(-h)^2 = -\frac{f^{(3)}(\xi_c)}{6} h^2$$

left



$$= \frac{f^{(3)}(\xi_a)}{3!} (1)(2)(-h^2) = \frac{f^{(3)}(\xi_a)}{3} h^2$$

right



$$= \frac{f^{(3)}(\xi_b)}{3!} (-2)(-1)(-h)^2 = \frac{f^{(3)}(\xi_b)}{3} h^2$$

Let's a better method by appropriately
averaging two methods, e.g. forward &
backward two-point difference formulas:

$$f'(x_0) \approx \frac{f(x_0+h) - f(x_0) + f(x_0) - f(x_0-h)}{2h}$$

$$= \frac{f(x_0+h) - f(x_0-h)}{2h} + \frac{\frac{1}{2} \left(\frac{h}{2} f''(\xi_f) - \frac{h}{2} f''(\xi_b) \right)}{\frac{h}{4} (f''(\xi_a) - f''(\xi_c))}$$

$$\frac{h^2}{4} \frac{(\xi_f - \xi_b)}{h} \left(\frac{f''(\xi_f) - f''(\xi_b)}{\xi_f - \xi_b} \right)$$

$\underbrace{\hspace{10em}}_{f'''(\xi)}$