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MAT 360 Numerical Analysis

Prof. Long

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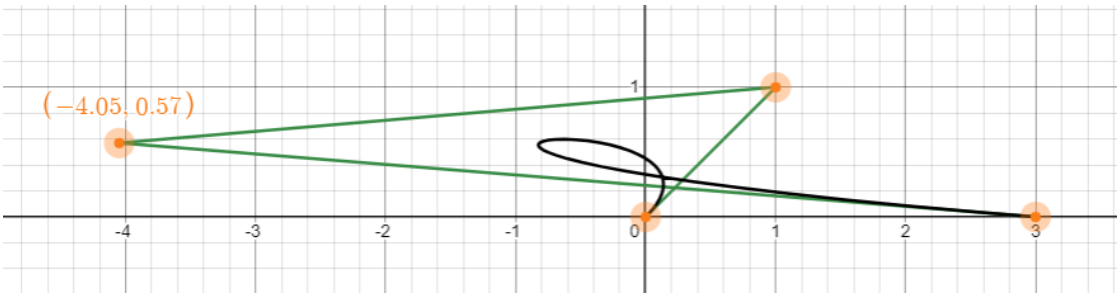
Bezier Cubic Spline SigMac Project

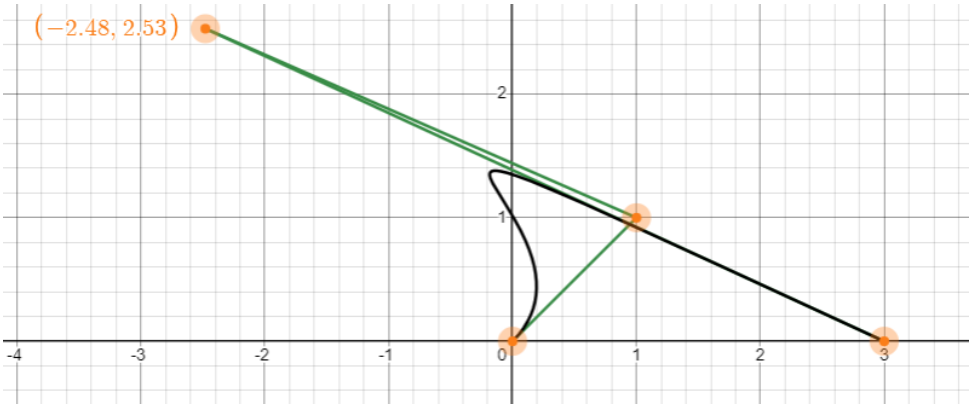
Question 1: Given points P_0=(0,0), P_1=(1,1), P_3=(3,0), investigate graphically the range of motion of the Bezier cubic possible depending on the choice of control point P_2.

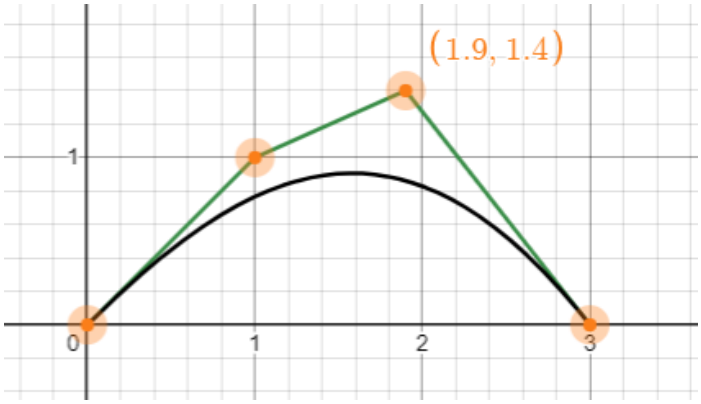
Describe the kinds of qualitatively different curves you can generate.

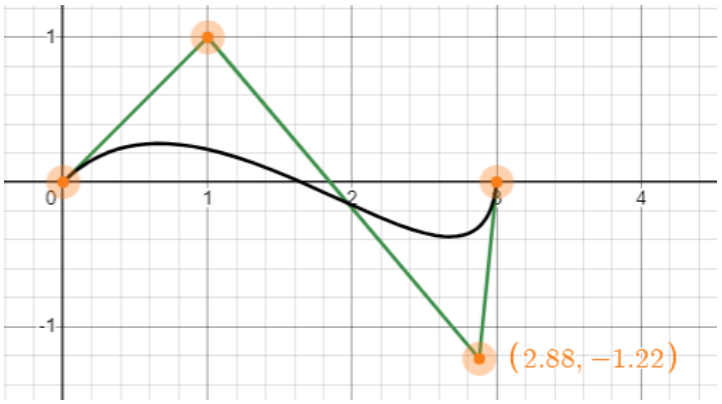


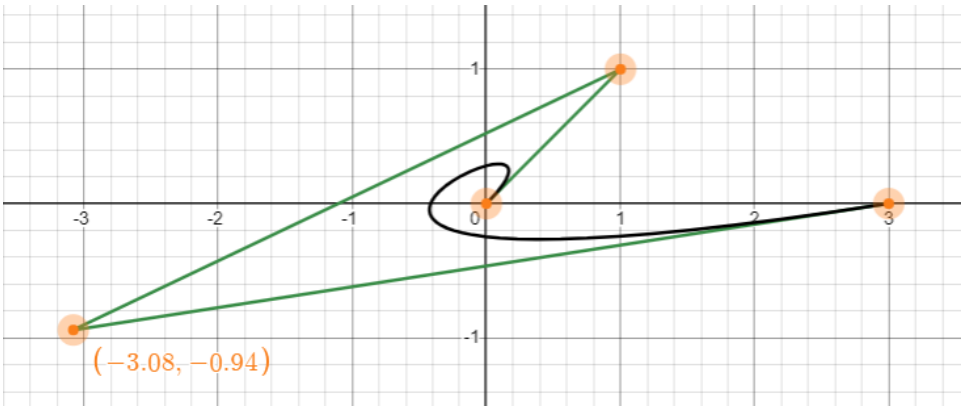
By moving P_2 behind P_0 and P_1, we can create loops.

When we place P_2 above P_0, P_1, and P_3, we create mountains of different slopes and shapes.







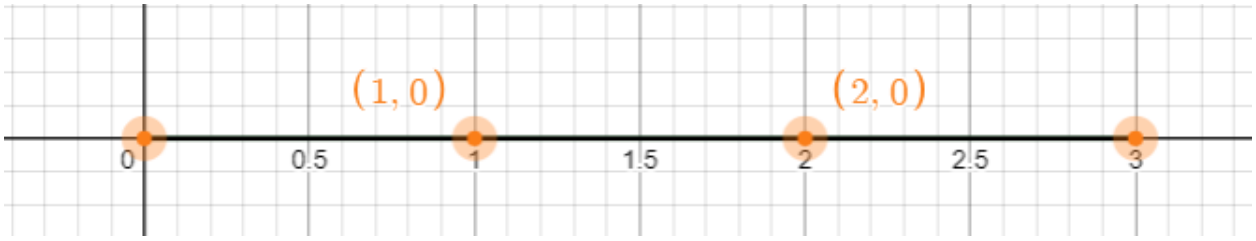


By placing P_2 below and/or behind P_0, P_1, and P_3, we can create all sorts of loops.

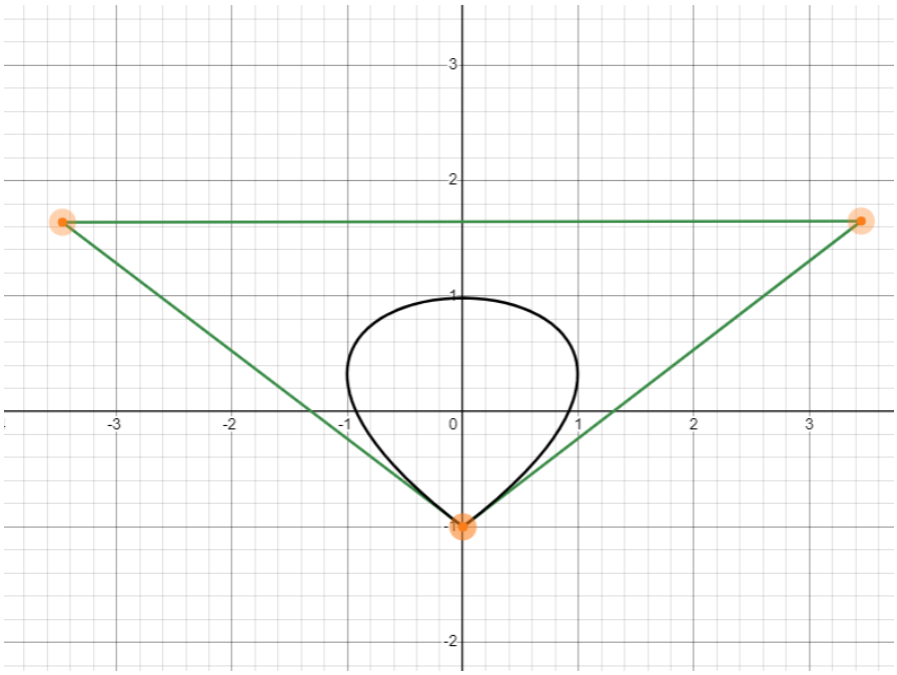
Question 2: Under what conditions will a Bezier cubic actually pass through one or both of the control points, P_1, and P_2?

There are a few ways we can have our Bezier cubic spline actually pass through one of our control points, as shown to the left.

The only way for our Bezier cubic spline will run through both control points P_1 and P_2 is if all four control points fit linearly.

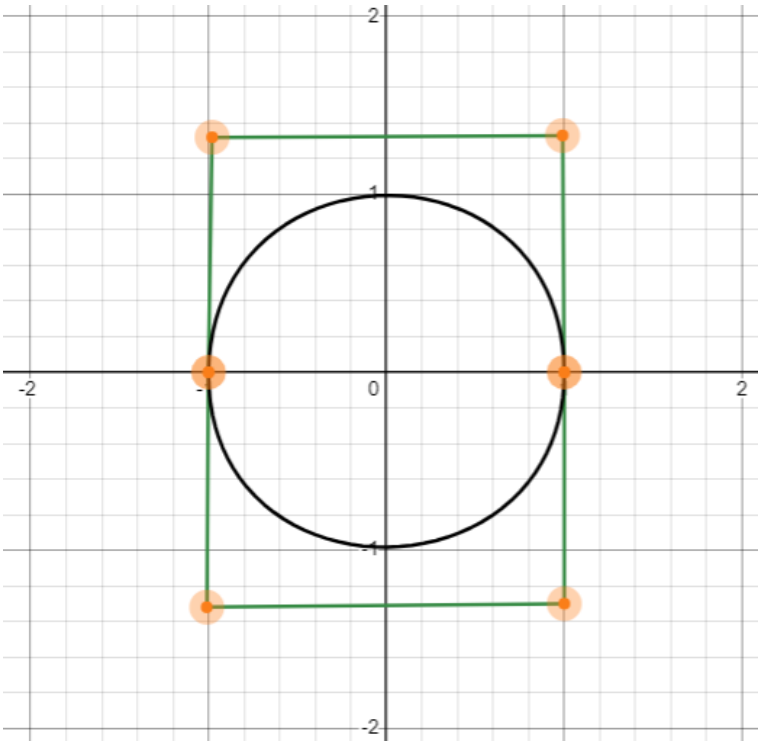


Question 3: Since many letters contain a circle, create a Bezier cubic spline that does a nice job of approximating the unit circle. Describe your criterion for "nice".



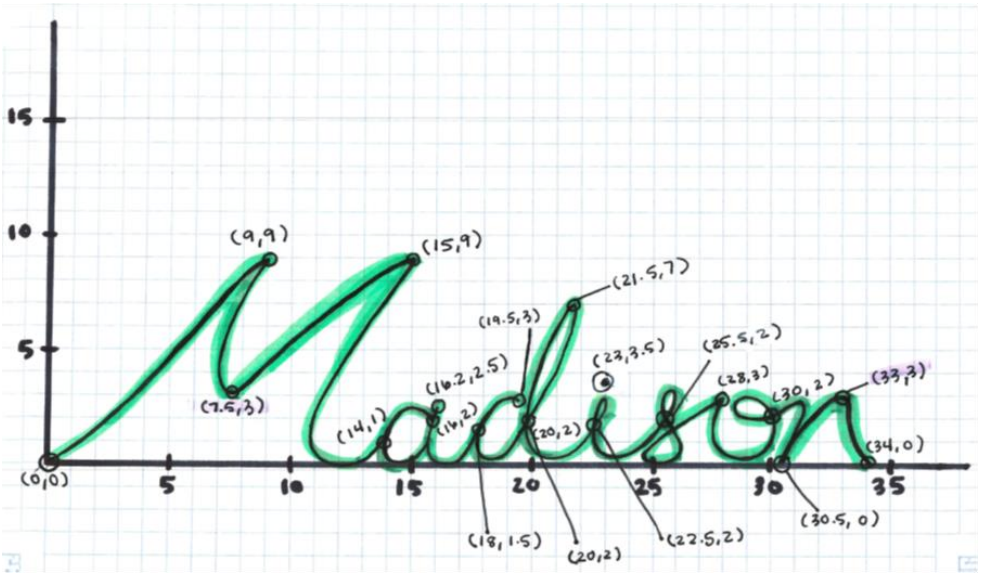
Creating the Unit Circle with a singular Bezier cubic spline proves to be a difficult task. No matter how you try to connect the two end control points to create a closed shape, there will always be a sharp point, making it difficult to create a circle. I used (0, -1) for both P_0 and P_3 to at least get that point from the unit circle "right." I then moved P_1 and P_2 around to fit the curve onto the point (0,1) to get that point "right." But we still miss (-1,0) and (1,0).

I then used two Bezier cubic splines to try to create the Unit Circle. For one spline, I used control points P_0=(-1,0), and P_3=(1,0) and then used P_1 and P_2 to fit the point (0,1) onto the curve. I then did the same thing, except in the negative direction and used P_1 and P_2 to fit (0,-1) onto the curve.

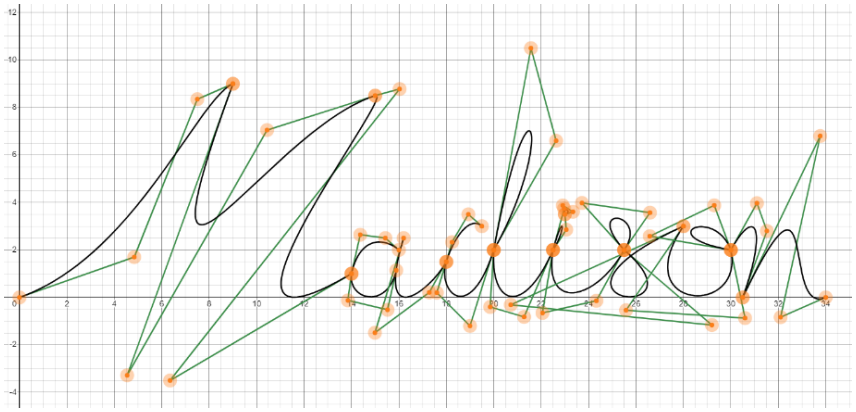


This was a "decent" or "nice" approximation of the Unit Circle, since we fit two points exactly and another two points at least visually very closely. However, the more splines we use to approximate the Unit Circle, the better the approximation. If we added another two Bezier splines, we could use all four points (-1,0), (0,1), (1,0), and (0, -1) as end control points and visually fit the rest of the curves to create the circle. This way we get another two points exactly "right." But this is true in general. The more splines we use and the more end control points we use from the actual unit circle, the more points we fit exactly and the better or "nicer" our approximation will be.

Question 4: Sample your signature for the interpolating points (best to do this on a piece of graph paper, so that you can get appropriate coordinates for control points), and then choose your control points so as to generate your sigmac. Print out a copy of your signature, as well as the output of your sigmac.



Here was the original sample of my signature. The points shown are the control points I decided to use. I used these for the end control points of the Bezier cubics and then used the other two control points of each spline to fit the curve to my signature. I also used a couple "test" points (highlighted in purple), not as control points, but just to compare and fit the curve to the signature.



Pictured above is the final SigMac for my signature.

Question 5: Discuss the success you had, and the problems and/or challenges you encountered.

One problem I encountered was having to adjust some of my initial control points. It wasn't by much, most of the time it was only by .5-1 units. Some of the curves just weren't lining up how I wanted them to, even by adjusting the middle two control points. So, there were a couple of end control points that I had to move around to get matching slopes with adjacent Bezier splines. But, in hindsight, it was a minor problem. It didn't happen every time and I kind of enjoyed messing around with the curves to get them to fit how I wanted them to. Another issue I encountered was having to add more splines than I initially thought, which meant I also had to define more control points. Some of the small, detailed spaces needed two or three splines in order to get the shape right. On the other hand, I was able to cover some large sections with just one spline, so it kind of evened out in the end. The only other thing issue wasn't really an issue, it was just tedious. It involved a lot more control points than I initially thought. It was very easy to get overwhelmed by the number of points and keeping track of which one is which. But once I got a system down it was basically just repeating the same process with each spline.