MAT112 Final, Fall 2004

Name:

Directions: Problem 7 is worth 20 pts; all other problems are worth 10. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts). Calculate the following integrals:

1.

$$\int_0^1 2x\,dx$$

2.

$$\int_1^e \frac{dt}{2t}$$

3.

$$\int e^{2x}dx$$

Problem 2 (10 pts). Compute the following integrals:

1.

$$\int_{3}^{5} \frac{4x-2}{(x^2-x)^2}dx$$

2.

$$\int \frac{1}{x(1+\ln(x))}dx$$

Problem 3 (10 pts). Use 5 equal subintervals to approximate the integral

$$\int_0^2 (2u - 4)du$$

Please use the midpoints of the intervals for your sample points.

Use your knowledge of geometry and a graph to verify that your answer in the first part is correct.

Problem 4 (10 pts). Let's see if you were paying attention in class.... An oil tanker is leaking oil at the rate given (in barrels per hour) by

$$L'(t) = \frac{80\ln(t+1)}{t+1}$$

where t is the time (in hours) after the tanker hits a hidden rock (when $t = 0$).

1. Find the total number of barrels that the ship will leak on the first day.

2. Find the total number of barrels that the ship will leak on the second day.

3. What is happening over the long run to the amount of oil leaked per day?

Problem 5 (10 pts). Use the definition of the derivative as a limit to find the derivative of the function $f(x) = \frac{1}{x}$.

Problem 6 (10 pts). Two banks are offering competing options for an investment loan: 3.25% quarterly, or 3% continuously.

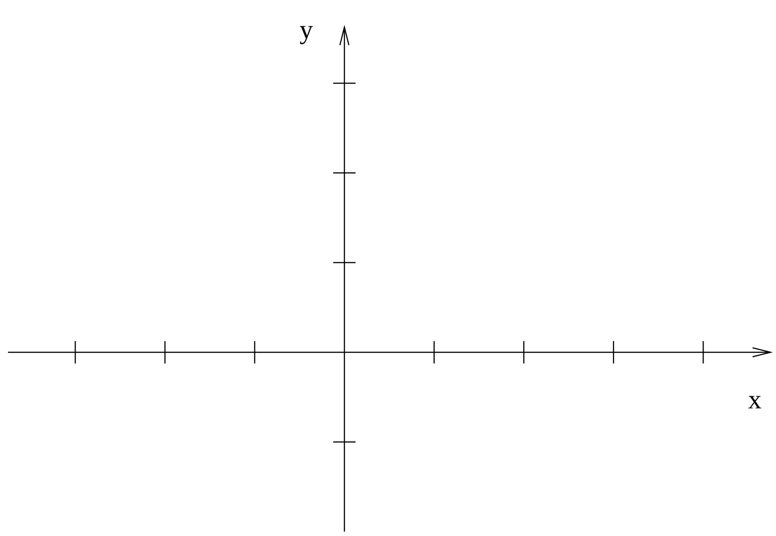
1. Which investment will give the best return? Explain!

2. Suppose you go with the continous compounding: how long will it take your investment to double?

Problem 7 (20 pts). Study the function

$$f(x) = \frac{x}{x^2 + 4}$$

on the interval $[-2, \infty]$, finding the domain; the intercepts; asymptotes, vertical and horizontal; the derivative; any critical points; the second derivative; relative and absolute extrema; determine where f is increasing and decreasing (in conjunction with first derivative); concavity and points of inflection. Finish with a plot.



Problem 8 (10 pts). Revenue from your production of toothpaste caps increases as the number of units sold increases, but subject to diminishing returns. Your staff models the revenue as $R(x) = 1000x^{1/3}$, where x is in thousands of caps, whereas the cost of making x caps is $C(x) = 20x$ in dollars.

1. Write the profit as a function of x.

2. What is the marginal profit when $x = 50$? When $x = 100$?

3. How many caps should your company produce in order to maximize profit? What is that maximum profit?

Problem 9 (10 pts). Solve or answer the following:

- $2^x = 100$

- $\ln(3^x) = 9$

- $\log_6(x) = 35$

- What is
$$\lim_{x \to 1} \frac{x-1}{x^2-1}?$$

- Where is $\frac{x-1}{x^2-1}$ continuous? Differentiable?