

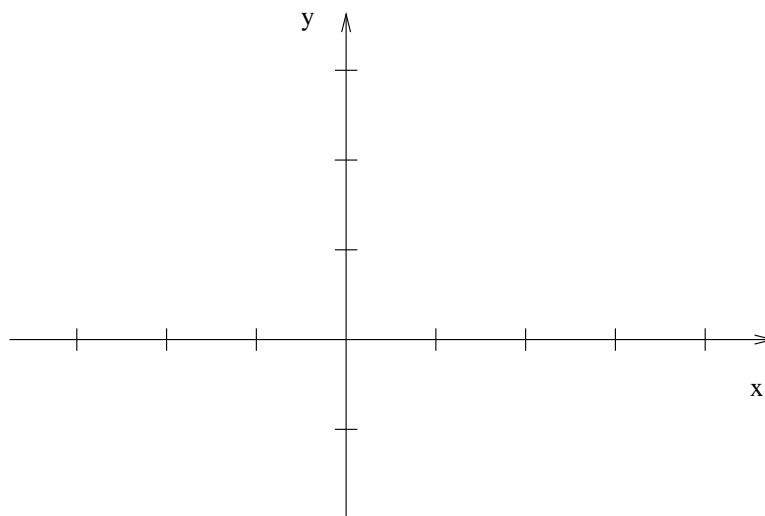
MAT112 Test 2: Limits and Derivatives

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Consider the function f defined by

$$f(x) = \begin{cases} 1 & x < 1 \\ (x-1)^2 + 1 & x \geq 1 \end{cases}$$



1. Carefully graph f on the axes above.
2. Find the limits (if they exist):

$$\lim_{x \rightarrow 1^+} f(x)$$

$$\lim_{x \rightarrow 1^-} f(x)$$

$$\lim_{x \rightarrow 1} f(x)$$

3. What is the derivative of f ? Does the derivative of f exist at $x = 1$? Why or why not?

Problem 2. Use the rules of derivatives to compute the derivatives of the following functions:

1. $u(y) = \frac{1-y}{y^2}$

2. $g(t) = 2t(3t - 1)^2$

Problem 3. Give an example of each of the following, or explain why it can't happen (you may draw a figure if that helps):

1. A function continuous at a point, but whose derivative does not exist there.

2. A function differentiable at a point, but not continuous there.

3. A function that fails to be differentiable in three distinctly different ways.

Problem 4. Someone asks you what a derivative is: what are three things about the derivative that you can tell them that will help them to understand?

1.

2.

3.

Problem 5. Use the definition of the derivative as a limit to find the derivative of the function x^2 .

Problem 6. The Revenue generated by your production of x widgets is $R(x) = \sqrt{x}$, while the cost of making x widgets is $C(x) = .1x$

1. Write the profit as a function of x .
2. What is the marginal profit when the number of units is $x = 25$?
3. How many widgets should your company produce in order to maximize profit?

Problem 7. Consider the function $f(x) = e^{x-1} \ln(x^2)$. Compute its derivative $f'(x)$, demonstrating the product rule and the chain rule as you do so.