Early Concepts of Number and Counting

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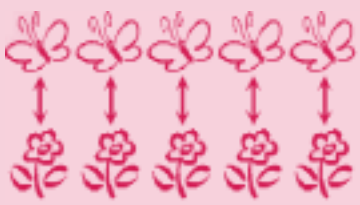
Introduction

Before primitive man had grasped the concept of number, the written word or even speech, he was able to count. This was important for keeping track of food supplies, sending messages, trading between villages and even keeping track of how many animals were in their herd. Counting was done in various ways, but in all cases, the underlying principle was one-to-one correspondence. This correspondence between the objects being counted and their counting aid, enabled primitive man to make the first important steps towards an abstract counting system.

Over time, and at varying places around the world different methods for counting arose. We survey here counting aids such as collections of pebbles, tally sticks and knotted strings, the use of the hand as a way of representing numbers, and counting with other parts of the body.

One-to-one correspondence

Below are two sets: a set of five butterflies and a set of five flowers. Between these sets, arrows indicate a 1–1 correspondence. Each butterfly corresponds to exactly one flower, and similarly, each flower corresponds to exactly one butterfly. So we have a 1–1 correspondence between the two sets.



Counting

Before we discuss how early man was able to count objects, we must first understand the concept of one-to-one correspondence. Primitive people would not have known of this concept; however, it clearly underlies primitive counting methods. Therefore we will use it here in this explanation of primitive counting.

One-to-one correspondence

In mathematical terms, a one-to-one (1–1) correspondence between sets S and T is a map such that distinct elements in S have distinct images in T; also each element of T is the image under the map of at least one element of S. More simply, each element in the first set corresponds to exactly one element in the second set and vice versa.

It is this 1–1 correspondence, or reckoning, that enabled primitive man to begin to count. Once man was able to see this correspondence between objects he was able to use it to keep track of anything of importance to him. For example, a shepherd could keep track of the number of sheep in the flock. While this was not an abstract counting system and the actual number of objects was not known, primitive people were able to tell if there were any objects added to or subtracted from the group. This was done by comparing the group with whatever was being used to keep count. Thus the shepherd with his sheep may have a pouch of pebbles by which he can tell if any sheep have gone astray. By making a correspondence between each pebble and each sheep, the shepherd can tell if any are missing without the use of numbers. This method also allows for the death, loss or sale of any sheep

by the removal of pebbles from the collection, and the birth of any lambs by the addition of pebbles to the collection.

Understanding 1–1 correspondence was the first step in the development of abstract counting. Instead of just recognising more or less when changes were made to a small group of objects, man now knew exactly how many objects were present.

This correspondence was used in many ways by primitive people and for many reasons, which will be discussed in the section to follow. Eventually this method of counting lead to the creation of number names and, in the end, number symbols.

Methods of counting and their uses

As mentioned earlier, primitive people had various ways in which they counted objects. In many cases, the tools used to keep track of various objects depended on the availability and ease of use of different materials. For example, some cultures used collections of shells while others used tally sticks. No matter what aid was used for the counting process, the underlying concept was the same. It was all about making a correspondence between the objects of interest and the counting aids. What is interesting about this type of counting is the different methods used.

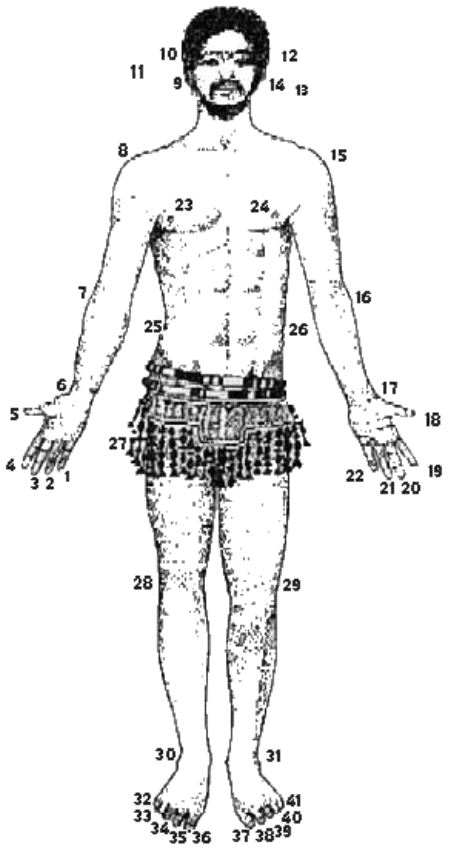
Body counting

A very common form of counting was by using the hand. The hand was an obvious choice for the use of correspondence counting, or reckoning, as it is an instantly accessible device. As noted by Aristotle, the widespread use of counting to ten is not the result of a calculated choice, but rather the anatomical accident that most humans are born with ten fingers and ten toes. So man learned to count using his fingers and perhaps at this stage, without names for the numbers that he was counting. Nevertheless, this was an effective method of counting that is still used all over the world today. Even in schools today, young children are taught to count on their fingers.

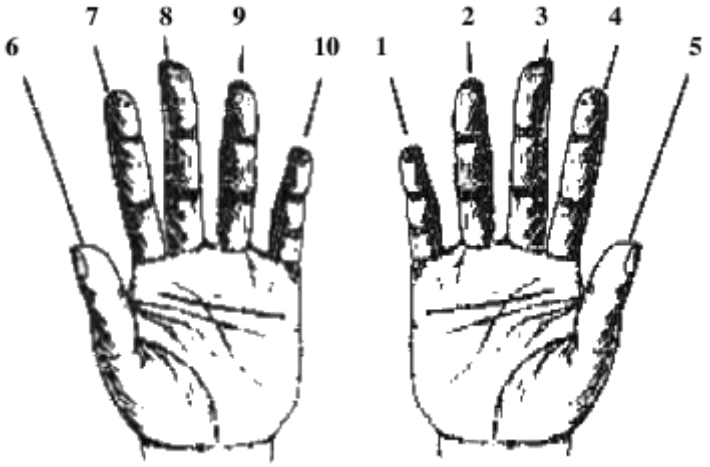
Finger counting was such an effective method that it was used by some people to count to numbers greater than ten. For example, there exist certain South African

Body counting

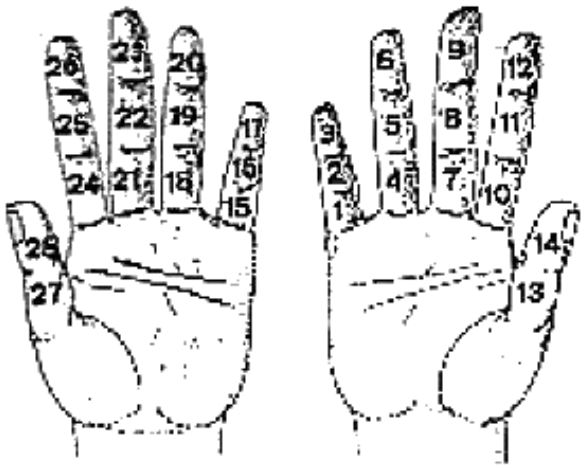
This idea of using a part of the body to count was taken to a different level by other cultures. People from places like Papua New Guinea and the Torres Straits Islands used their whole body to count. By taking various points of their body, for example including the eyes, nose, hips, etc., as well as the fingers, these people were able to count to numbers as high as 41. This diagram illustrates one particular method of body counting.



1. Right hand little finger
2. Right hand ring finger
3. Right hand middle finger
4. Right hand index finger
5. Right thumb
6. Right wrist
7. Right elbow
8. Right shoulder
9. Right ear
10. Right eye
11. Nose
12. Mouth
13. Left eye
14. Left ear
15. Left shoulder
16. Left elbow
17. Left wrist
18. Left thumb
19. Left hand index finger
20. Left hand middle finger
21. Left hand ring finger
22. Left hand little finger
23. Right breast
24. Left breast
25. Right hip
26. Left hip
27. Genitals
28. Right knee
29. Left knee
30. Right ankle
31. Left ankle
32. Right foot little toe
33. Next toe
34. Next toe
35. Next toe
36. Right foot big toe
37. Left foot big toe
38. Next toe
39. Next toe
40. Next toe
41. Left foot little toe



Simple finger counting



Extended finger counting

people that employ the use of three people to count to numbers larger than one hundred. This is done by the first person, beginning with the little finger on their left hand, counting continuously until ten is reached. Then when the first person reaches ten, the second person raises their left little finger and the first person starts again. Each time ten is reached, the second person raises a finger. Similarly, when one hundred is reached the third person raises their left little finger and the process continues until the desired number is reached. Therefore, by using three people, the first representing units, the second representing tens and the third representing hundreds, these South African people were able to count to numbers above one hundred.

The idea of counting on the fingers was extended by some cultures to reach numbers higher than ten. For example, a method used in China and India enabled a person to count to 28 using the finger joints on each hand. This can be done as illustrated above.

Using the body as a counting tool, people were able to communicate information between tribes. For example, a messenger could be sent to another tribe with the message that they wish to trade 20 baskets of food for 15 pearl necklaces, say. This could have been done by simply indicating the point on his body that corresponded to the correct number of objects.

Unfortunately, counting on any part of the body was not permanent and in many cases permanent records needed to be kept. So while body counting was extremely useful, another method would be needed if any type of permanent record were to be kept.

Collection counting

Another type of method of keeping a record of the number of objects is by having a collection of smaller objects that can be used to make a 1–1 correspondence with the objects to be counted. This method was briefly explained earlier with the example of the shepherd, his sheep and a collection of pebbles. Keeping a record of the number of objects to be counted is as simple as placing one of the smaller objects in a pile for each of the objects to be counted. In this way, as explained earlier, there is one smaller object for each of the counted objects. Now if we were to come back and check our counted objects, all that is required is to repeat this method. It can quickly be seen if there are any objects missing, as there would be some of the smaller objects left over.

A slight variation on the example of the shepherd is the following. For each animal let out of the enclosure, place one pebble in a collection. Then at the end of the day, remove a pebble for each animal that returns to the enclosure. This way you quickly see if you have lost or gained any animals throughout the day.

Many primitive people in varying ways used this method of keeping tally. Some cultures used collections of shells, pearls, pebbles, elephant teeth, sticks, or even coconuts to keep a tally. These were quite useful methods that were employed frequently by early people, however, it could be quite impractical in some cases. Say the shepherd had a very large flock of sheep; it would not be very convenient to be carrying around a large, heavy bag of pebbles. So from this we move to another method of keeping a tally of objects.

Tally sticks

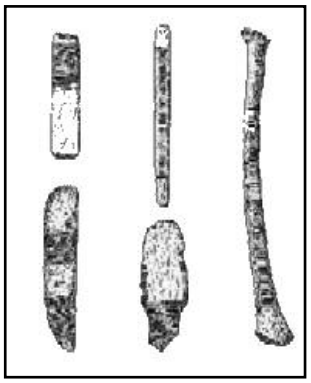
Tally sticks are yet another way of counting a group of objects. They consist of a piece of wood, bone or any object in which it is possible to cut small notches. Each notch in the tally stick corresponds to an object being counted. Primitive man sometimes may have used this method of counting to make a record of anything of interest or importance, for example, the number of animals slain by a hunter. Tally sticks were a much more efficient way of keeping track of the number of objects being counted as one need only carry around a small piece of wood or bone. The adjacent picture illustrates some very primitive tally sticks made from bone.

As tally sticks were an extremely convenient way of keeping tally, they were still in use much later in time. However, in some cases, the tally sticks evolved to become more than just notches cut into wood and had more varying uses. Not only could they be used to count a group of objects, but they could also be used to keep a record of debts or credits owed by a person, the number of days, weeks, months or years gone by, or even a mark of ownership.

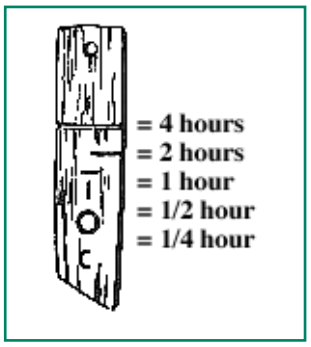
An example of a tally stick that contained numerical symbols, rather than notches, is the Swiss water tally. These tally sticks showed the amount of time to which a particular farming family had water rights on any given day.

Tally sticks could also be used to keep a record of credits. For example, if a baker had given a number of loaves of bread to a woman over a week and was to settle the account at the end of the week, then a tally could be kept in the following way. The baker would have one tally stick and the woman another. When a loaf of bread was received, the tally sticks would be placed together and notches would be cut in both, corresponding to the number of loaves (in the presence of both parties). This would continue during the week and at the end of the week the account could be settled without argument. No dispute would occur as the tally sticks would be placed together and compared. If the baker tried to add notches it would be obvious and it would be very difficult for the woman to remove any notches, therefore there would be no dispute about the account.

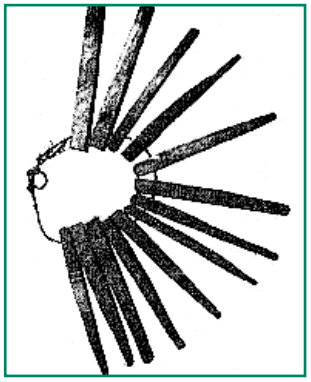
Tally sticks could be used for several customers and in many situations. Different tally sticks would be marked with the owner's mark, or signature. With the use of an identifying mark, tally sticks could be used for more accounting type purposes. Even until the 1800s, tally sticks were in use for taxation purposes. Shown on the right is a Russian 'tax book'. Each tally stick in the collection contains a family mark, or signature, the number of family members and the amount of tax to be paid.



Primitive tally sticks



Swiss water tally



Russian tax book

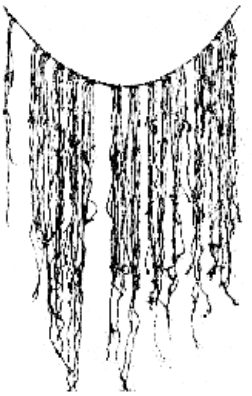
Knotted strings

We now move on to another type of counting that involves the use of knotted strings. In their simplest form, knotted number strings are much the same as the simple tally sticks. That is, one knot in the string has the same meaning as one notch in the tally stick. Counting with the use of knotted strings has been found all over the world. For example, Tibetan prayer-strings and the rosary are both simple knotted number strings, which are used to prescribe the number of religious exercises to be undertaken.

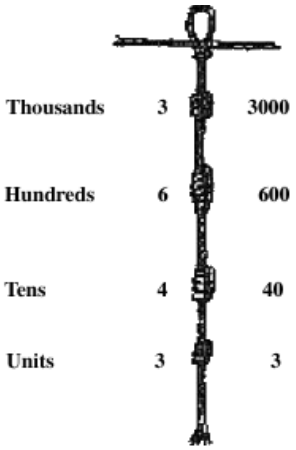
Like the tally sticks, the knotted number strings gradually evolved into a slightly more complicated device. Combinations and different types of knots replaced the simple single knot. Different types of knots carried different values and the relative positions of the knots on the string came to denote the order of magnitude of the number.

This can be seen clearly in the case of the Peruvian quipu, a number string used by the Incas to record, not only numbers, but also financial transactions, laws and even history.

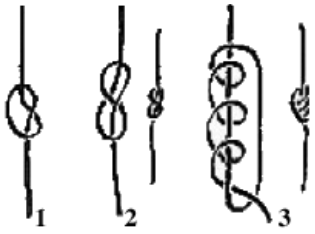
Each string coming from the main strand at the top contains knots which represent a particular number.



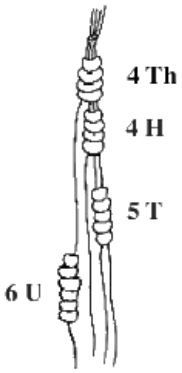
Peruvian quipu



The example 3643



Use of different knots



The chimpu

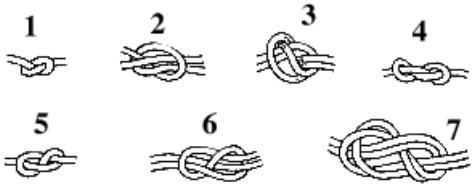
The knots have been grouped together along the string in such a way that the higher orders of magnitude are above lower orders of magnitude. That is, higher orders of magnitude are closer to the main strand.

For example, if the number 3643 were to be represented on one of these strands, it would resemble the illustration on the left.

Another detail that has been added to these quipus is the use of three different types of knots. The image alongside shows these different knots. First there was the single knot (illustrated as number 1), then the double or figure-of-eight knot (number 2) and finally the slip knots, which could have from 2 to 9 loops (number 3 with three loops). However, the double knot and slip knots were used only to represent the units in a representation.

Possibly a descendant of the quipu was the chimpu of the Bolivian and Peruvian Indians. The chimpu was a very similar device to the quipu, however, in this case numbers were represented using more than one string. The order of magnitude of a group of knots was not only determined by the relative position of the knots on the strings, but also by the number of strings the knots held together. For example, the number 4456 would be represented as shown in the illustration.

Knotted strings could be used to record any number of things such as taxes, wages owed, calculations and even the weights and types of bagged produce. In Germany, millers had their own system of knots that were used to inform the bakers of the weight and type of flour in a bag. These knots were tied in the ends of the strings that were used to seal the bags of flour. The knots shown above were used to represent the weights of the bags of flour.

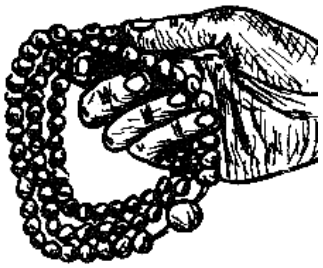


The measure of flour was the Sester which contained 10 Massel.

1. 1 Massel
2. 2 Massel
3. 5 Massel = 1/2 Sester
4. 10 Massel = 1 Sester
5. Alternative to 4
6. 2 Sester
7. 6 Sester

Weights of flour

A slight variation on the knotted number strings is strings of beads. Strings of beads can most commonly be found used for religious purposes, for example Christian rosaries. Even today, many religious persons such as Muslims, Buddhists and Catholics use prayer beads to help in the recitation of certain prayers. These strings of beads consist of the correct number of small beads and in some cases larger or coloured beads separate these smaller beads as markers. As recitation begins one moves their fingers over each bead as the prayers are said until no beads remain.



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