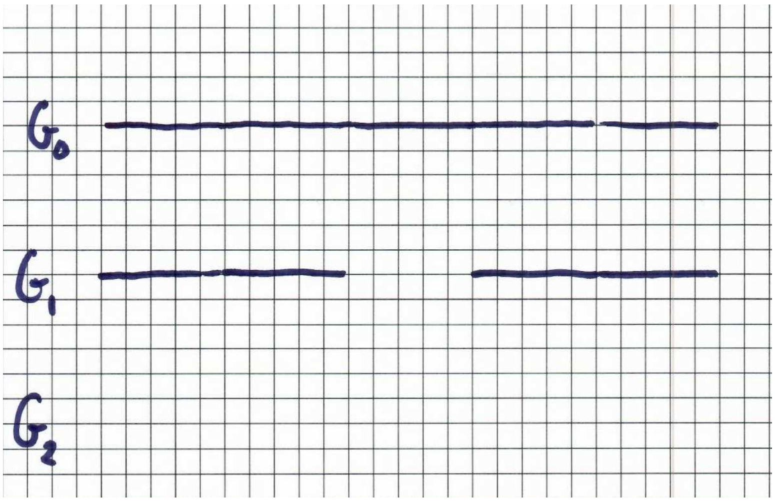
MAT115 Exam 2 (Spring 2025)

Name:

Directions: Show your work! Answers without justification will likely result in few points, and I can't provide partial credit. Indicate clearly your answer to each problem (e.g., put a box around it). Good luck!

Problem 1: (18 pts) Consider this stick of (red) licorice of length 1: at each generation, one nibbles the middle **fifth** from each stick of licorice in the previous generation.



a. (10 pts) Add the next stage of the fractal (generation 2, or G_2) to the figure above.

b. (2 pts) How is the fractal's total length changing from step to step?

c. (2 pts) How is the number of sticks increasing with each round?

d. (2 pts) How many sticks will there be at generation G_3?

e. (2 pts) What will be the length of licorice at generation G_3?

Problem 2: (20 pts) Symmetry:

a. (9 pts) Below each local logo, describe the symmetry you observe.

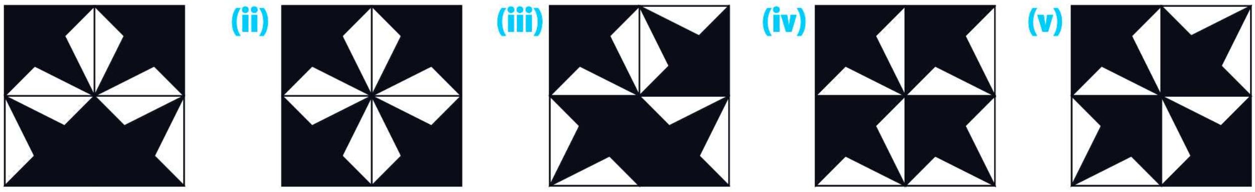
b. (11 pts) Carry out this exercise:

T2 Judith has lots of tiles, all like this one.

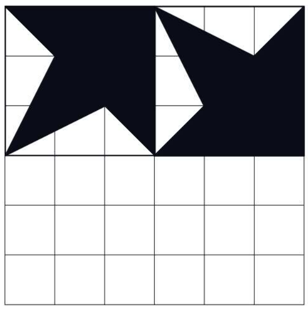
(a) Judith makes these patterns.

For each pattern, write down the number of lines of symmetry it has.
If the pattern does not have reflection symmetry, write 0.

(i) **(ii)** **(iii)** **(iv)** **(v)**



(b) Copy and complete this tiling pattern so that it has rotation symmetry of order 4.



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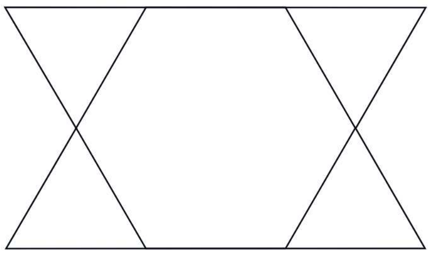
T3 This design is made from a regular hexagon and four equilateral triangles.

(a) What is its order of rotation symmetry?

(b) Write down the number of lines of symmetry it has.



Problem 3: (16 pts) Short answer:

a. What does it mean if I say that the graph of Facebook relations between five particular people is a complete simple graph?

b. There were two separate themes of your reading "The Enemy of my Enemy is my Friend": one about numbers and another about war. Describe the two themes.

c. If there is an Euler path in a connected graph, how many vertices may have odd degrees? (Give a complete answer: it's an either/or, not just a single number.)

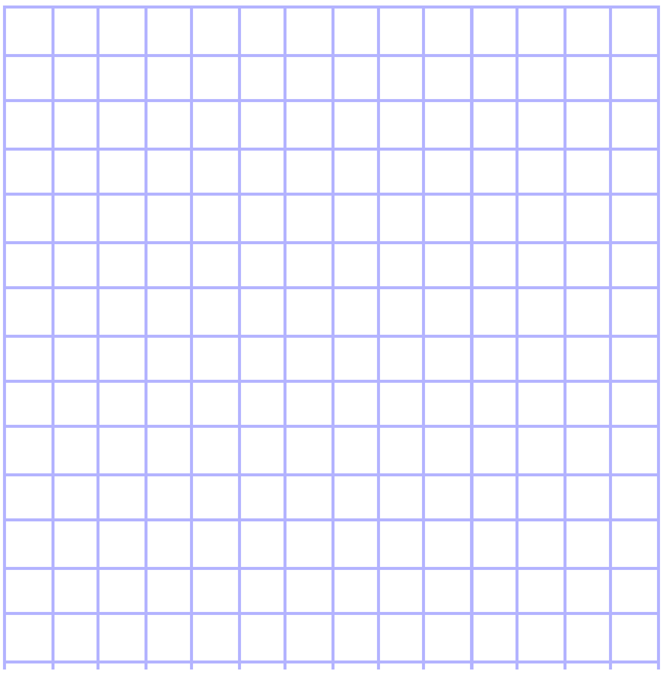
d. Given the following rectangles, which one is closest to golden? Give evidence....

   i. A 3x5 card
   ii. A sheet of $8\frac{1}{2}$ by 11 paper
   iii. A flag of dimensions 4 feet by 6 feet
   iv. A television screen of size 30x48 inches?

Problem 4: (14 pts) The golden ratio ϕ was discovered in class in two different ways: from ratios of Fibonacci numbers, and by chopping up a beautiful rectangle and using the quadratic formula.

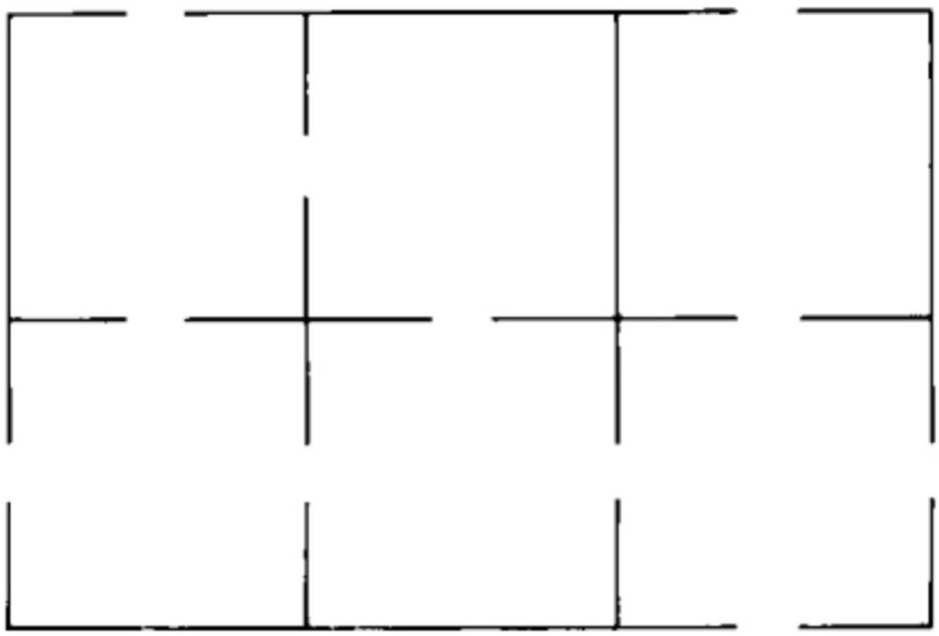
a. (4 pts) What was the Greek's definition of a golden rectangle, and how does it relate to ϕ? **Note:** you do not need to **derive** ϕ: just explain how the Greek's went about defining it (perhaps with a diagram).

b. (6 pts) In the grid provided, make the largest Fibonacci spiral possible. At right, describe the connection between this spiral process and the golden ratio and rectangle.



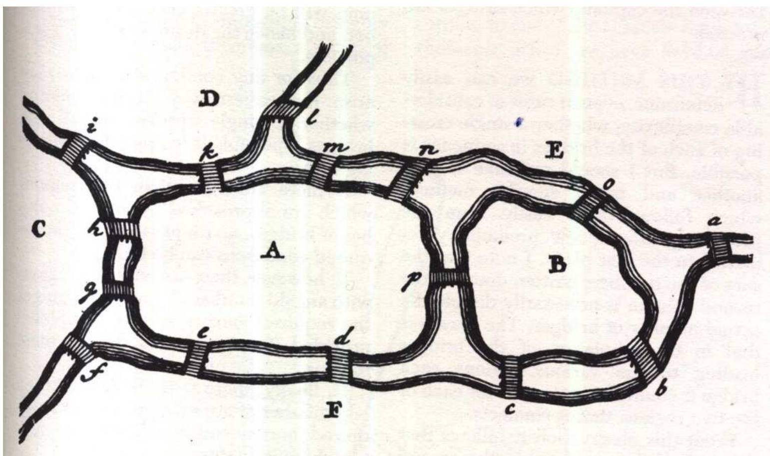
c. (4 pts) At each stage, compute the ratio of the largest to the smallest side. How close to golden are these rectangles?

Problem 5: (16 pts) The residents of a house play a game, in which they try to walk in and out of each room in their house (shown) so that each door is used exactly once:



a. Is it possible? If so, trace a path through the house, indicating where you start, and where you end; if not, indicate why it is impossible.

b. Euler considered another example of geography in his paper about Konigsberg, as shown in the figure below:



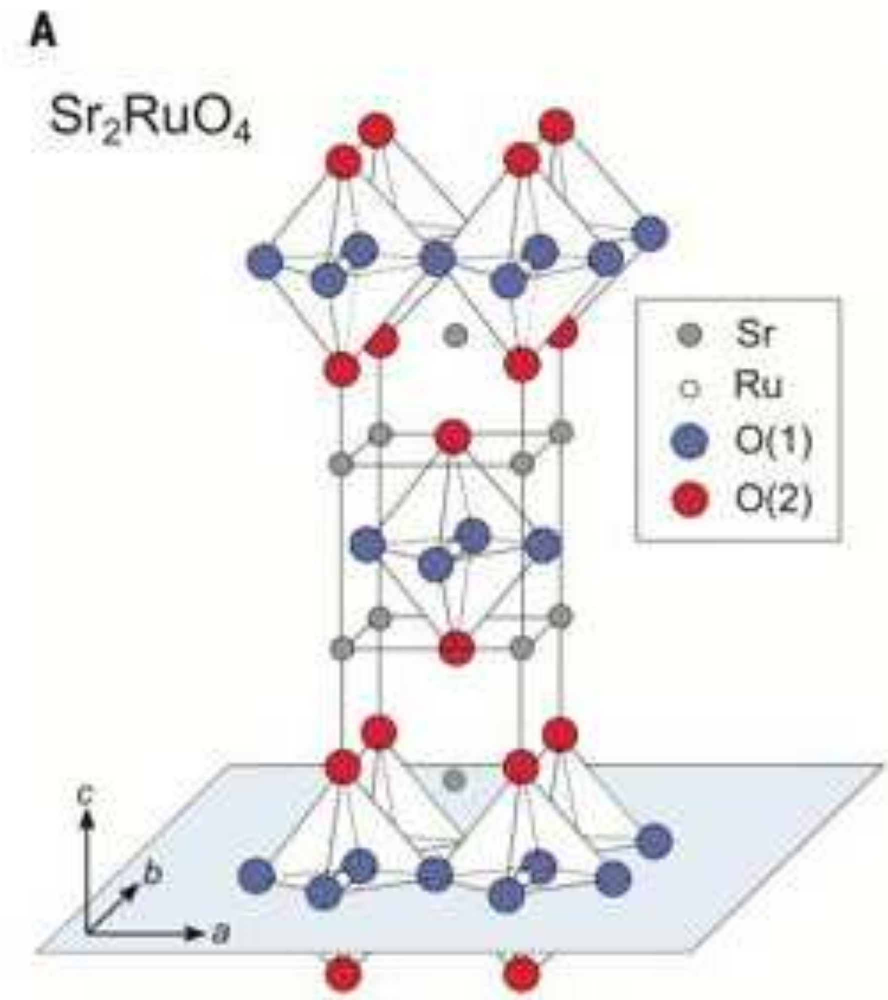
This trip is possible though the Koenigsberg one is not

i. Draw a graph that corresponds to this map. It's a planar graph, with 6 nodes and 15 arcs: how many regions does this graph divide the area into?

ii. Is an Euler path possible on this graph?

Problem 6: (16 points)

a. (10 pts) Point out as many connections as you can to the Platonic solids in the molecule Sr_2RuO_4



b. (6 pts) As accurately as possible, draw the dual Platonic solid into this cube (or any other cube of your choosing!):