EULER'S CRAZY CALCULATION

Euler started with this:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

He differentiated both sides:

$$1 + 2x + 3x^2 + \cdots = \frac{1}{(1-x)^2}$$

He set $x = -1$ and got this:

$$1 - 2 + 3 - 4 + \cdots = \frac{1}{4}$$

Then Euler considered this function:

$$\zeta(s) \;=\; 1^{-s} + 2^{-s} + 3^{-s} + 4^{-s} + \cdots$$

He multiplied by 2^{-s}:

$$2^{-s}\zeta(s) \;=\; 2^{-s} + 4^{-s} + 6^{-s} + 8^{-s} + \cdots$$

Then he subtracted twice the second equation from the first:

$$(1 - 2 \cdot 2^{-s})\zeta(s) \;=\; 1^{-s} - 2^{-s} + 3^{-s} - 4^{-s} + \cdots$$

Taking this result:

$$(1 - 2 \cdot 2^{-s})\zeta(s) \;=\; 1^{-s} - 2^{-s} + 3^{-s} - 4^{-s} + \cdots$$

and setting $s = -1$, he got:

$$-3(1 + 2 + 3 + 4 + \cdots) = 1 - 2 + 3 - 4 + \cdots$$

Since he already knew the right-hand side equals 1/4, he concluded:

$$1 + 2 + 3 + 4 + \cdots \;=\; -\frac{1}{12}$$