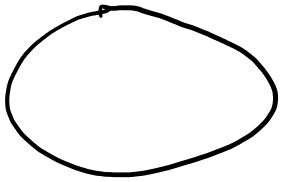
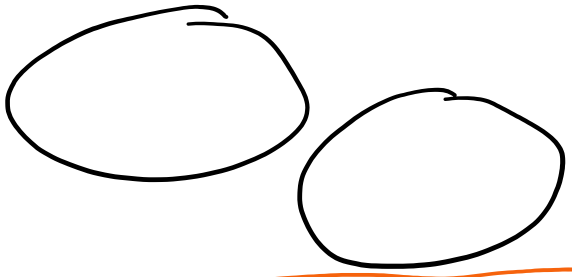
A Gallery of Some

Knots & Links

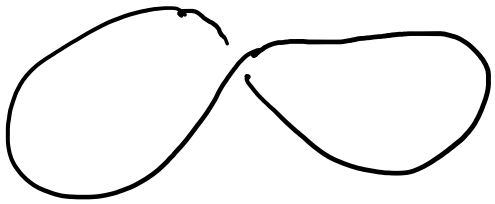
unknot



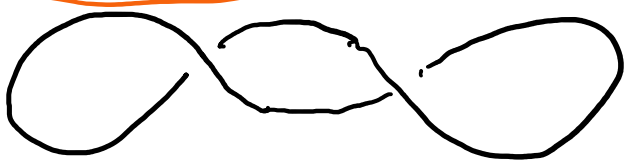
unlink



one crossing



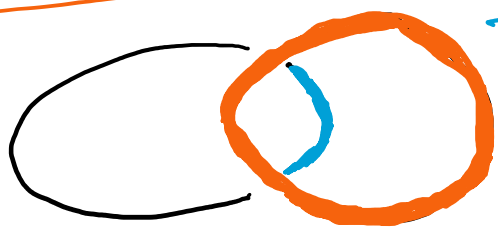
(not a new knot - this is in fact an example of a Reidemeister move - RI, & so still the unknot).



This two-crossing is just the unknot after two RIs.

There is no new two-crossing knot. A knot with one or two-crossings

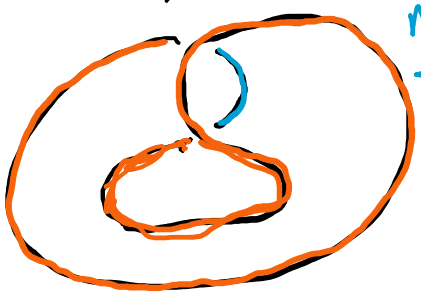
A one-crossing link is the unlink:



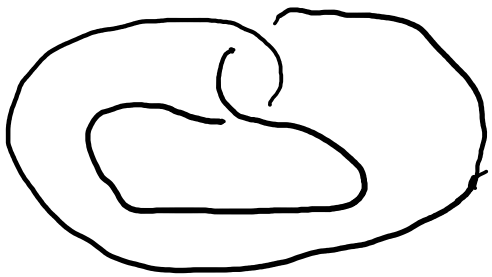
This two-crossing is an example of our second Reidemeister move - RII - & means that this is really the same as the unlink.

Here's a new link, w/ two crossings:

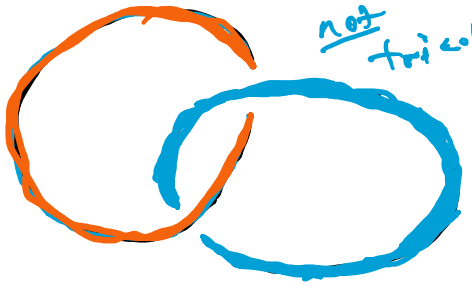
is just the unknot in disguise!



This two crossing knot (still the unknot!) can be thought of as an example of an RII or two RIs! Can you see that?



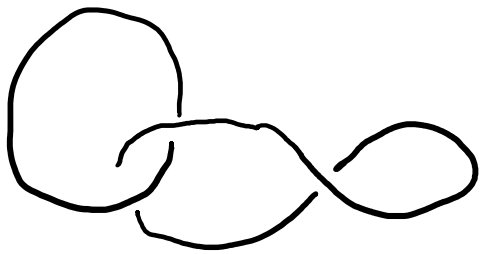
This one seems different, but it, too, can be thought of as two RIs from the unknot.



It's called the Hopf link.

There are no new links w/ 3 crossings,

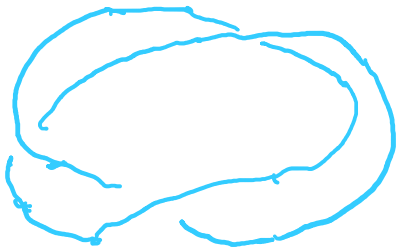
You can always add an RI to a Hopf link, e.g.:



But it's just a Hopf link w/ a twist!

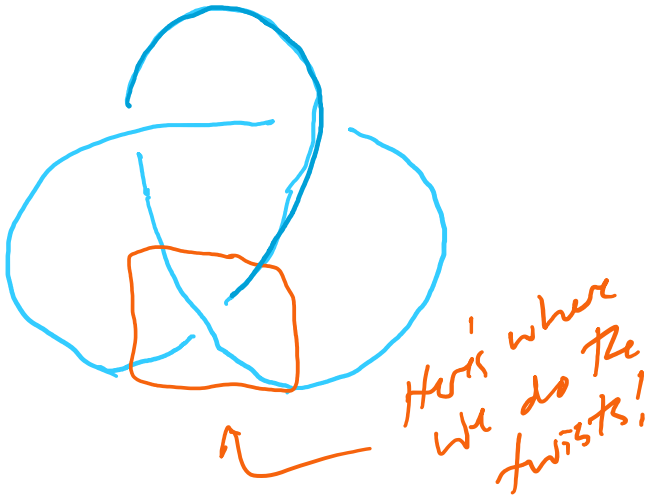
If we allow 4 crossings, however,

There's a new knot in town when we get to 3 crossings: the trefoil knot. And we can think of it as a torus knot,



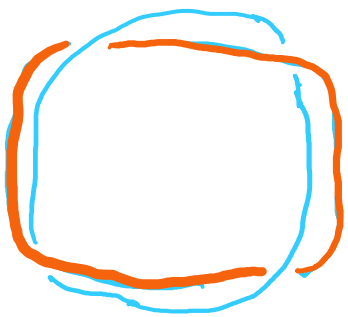
(a knot wrapped around a donut); or as the edge of a thrice-twisted band.

The other way to think of it is as a twist knot:

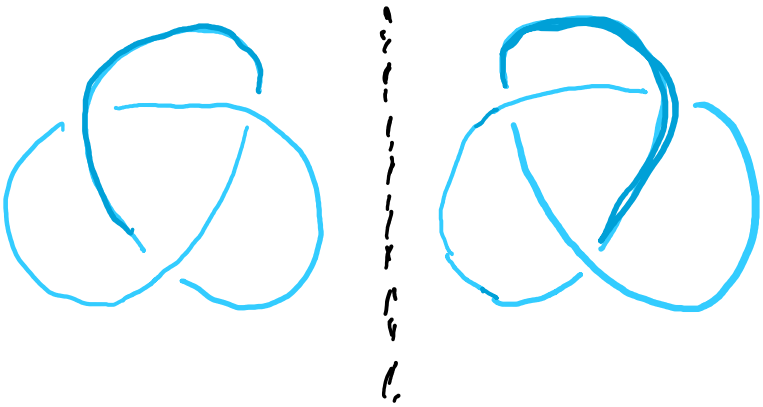


we get a new link: Solomon's "knot" (remember: this is actually a link!).

If we do the torus thing but give four twists instead of three, we get the Solomon's knot:



Note that there's a left-handed & right handed version of the trefoil; they're mirror reflections of each other!

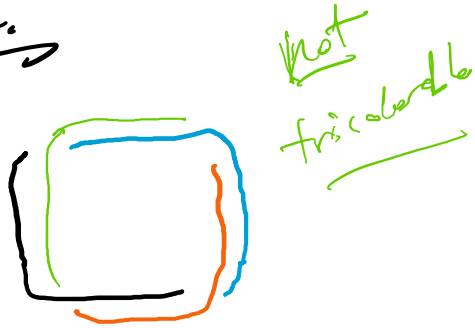


It turns out that it's impossible to transform one into the other w/o cutting it!

This notion has been used to try to create "symmetric sugars" that are harder for your body to absorb (because they have the wrong "handedness", etc.),

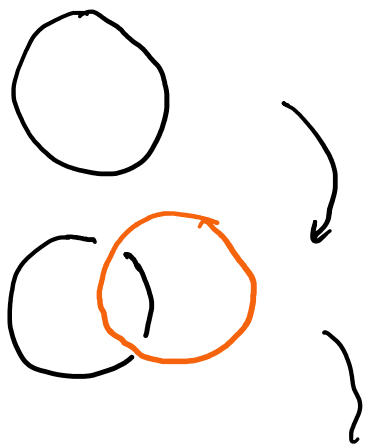
This knot is tricolorable.

We see that when we make four twists, we end up w/ two separate strings — a link.

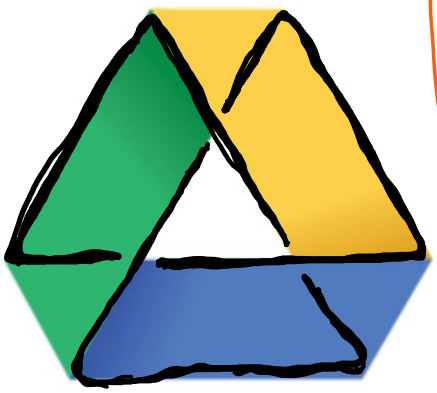


The last link I expect you to know is my favorite: the Borromean Rings!

It's three rings (not two, like the other links).



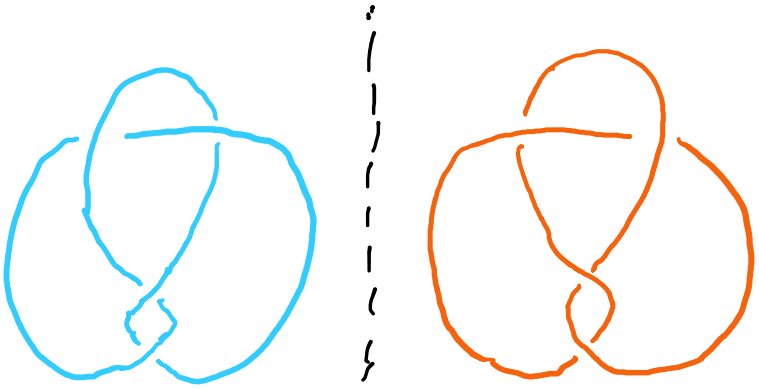
Notice that the Google Drive logo is a thrice-twisted band; its edge forms a trefoil knot:



(See the three leaves?)

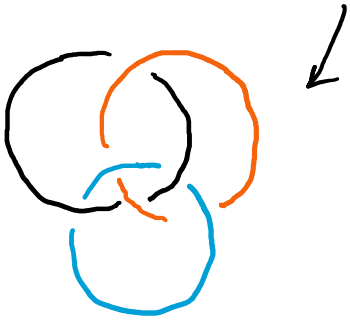
etc.

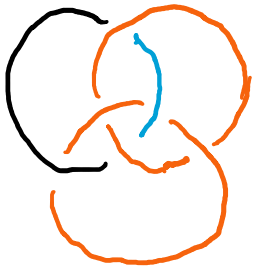
There is a four knot, but not a four-torus knot (that's the link called Solomon's knot); rather a four "twist knot":



We learned that this one

The "Figure Eight" knot also has left- + right-handed versions



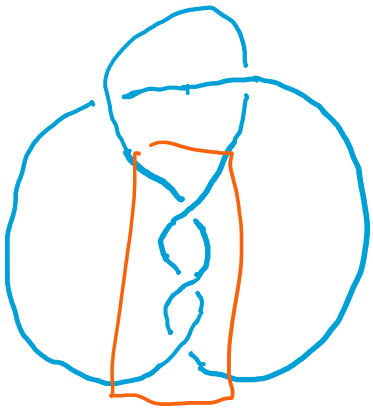


not tri-colorable

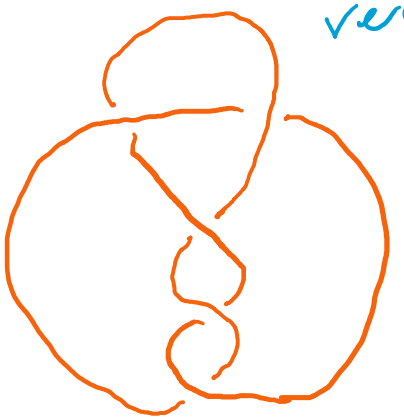
isn't tri-colorable,

There are two five (crossing) knots:

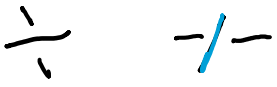
One more twist:



Left & right handed versions



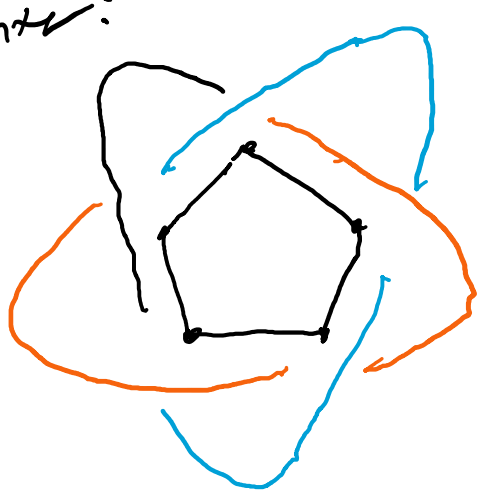
Crossing guide: overs to unders, unders to overs.



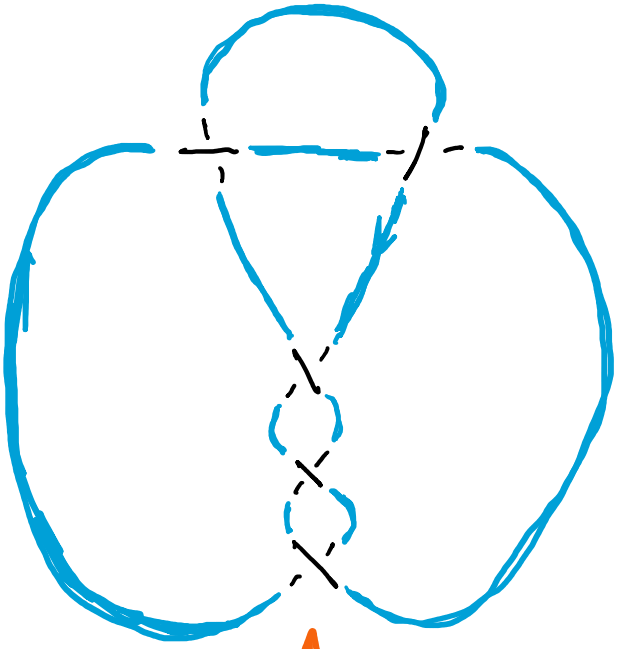
Then:
Just follow your nose!

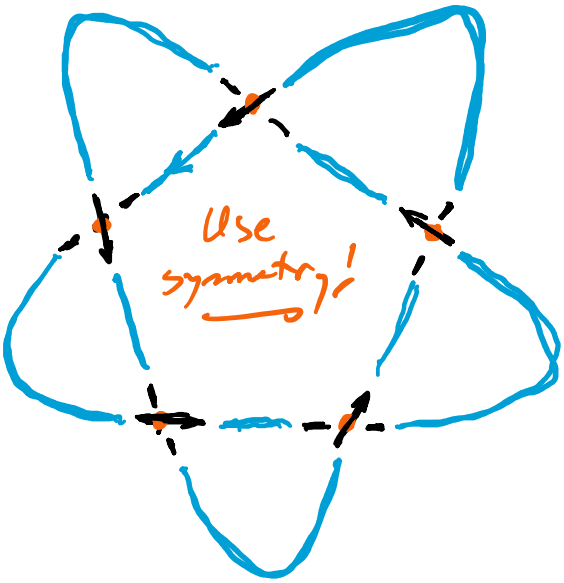
And a five torus knot:

Notice the pentagon at the center:



You can use that as an organizing tool!



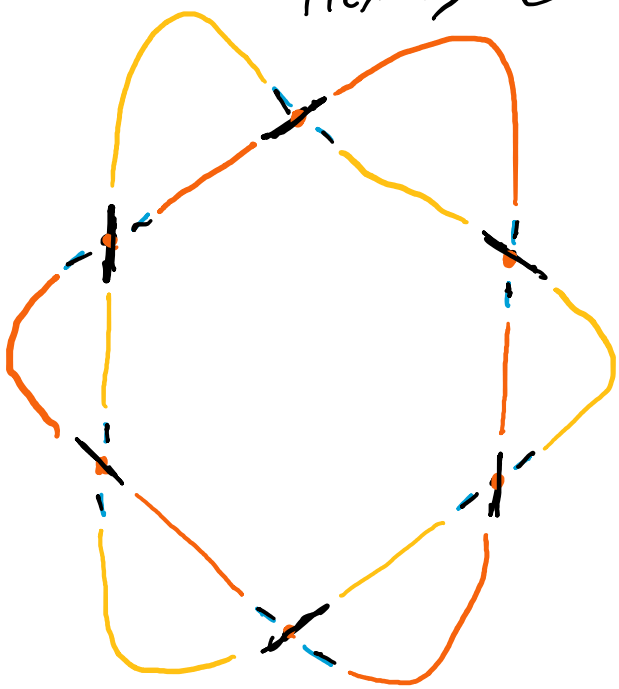


obviously we could construct knots with lots more twists!

Hexagon, Septagon, Octagon, ...

But the even ones give rise to links!

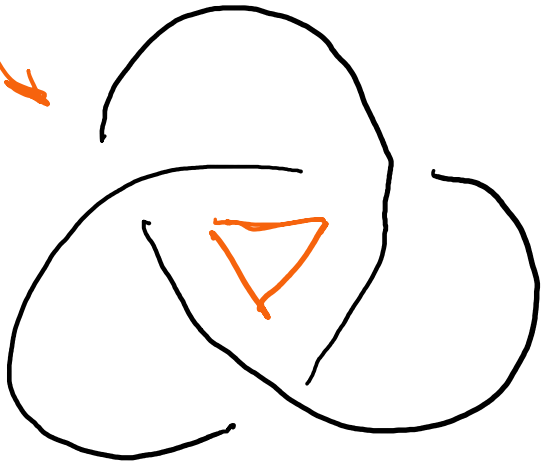
Hexagon



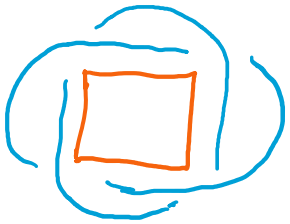
(Order 3) "Rotated fish hooks:"

(Torus Knots)

Trefoil



Order 4: Solomon's "knot"



Order 8: It's a link!

Note:

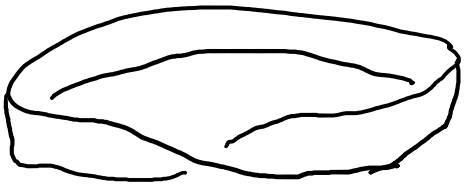
No two-sided or one-sided polygons —

No two- or one-crossing knots.

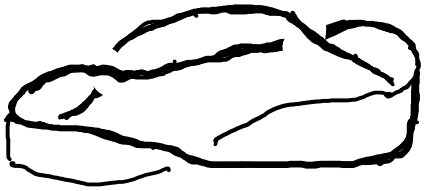
Final note: bands

Möbius

Thrice Twisted

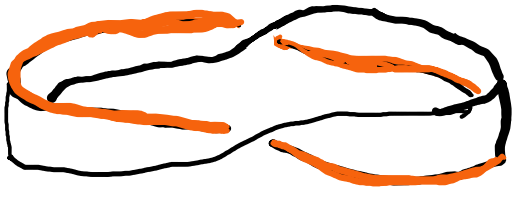


unknot edge



trefoil edge

Twice-Twisted



Hopf Link