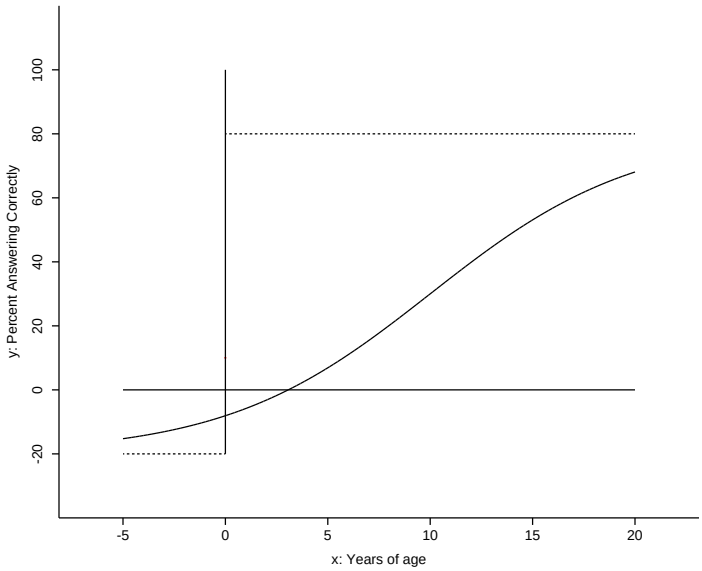
MAT120 Final: Spring 2003

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

There are twelve 10 point problems. You must skip two - write "skip" on them. If you fail to do so, I will skip two *at random* - so it's better that you choose!

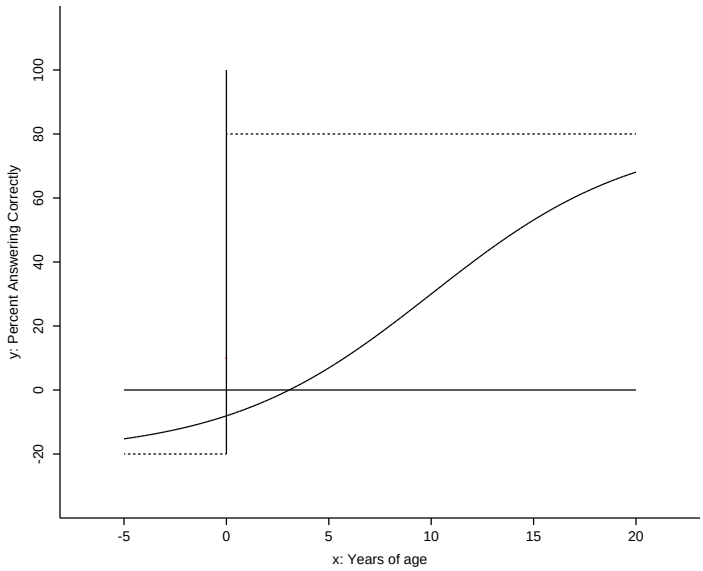
Problem 1 (10 pts). A test question to appear on a national exam is tested in a large school system. Students of all ages are given the questions, and the percent of correct responses as a function of age is modelled as in the following figure:



1. Draw in the first derivative, making note of any symmetry. Don't worry about scale; worry about shape.

2. Draw in the second derivative, making note of any symmetry.

Problem 2 (10 pts). Consider again the graph of Problem 1:



1. Of what significance are the intercepts, the inflection point, and the asymptotes?

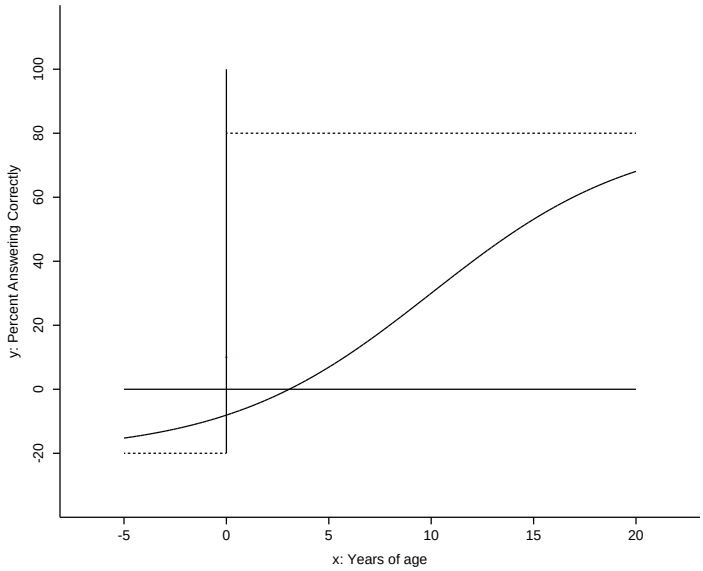
2. You are an employee of the testing company, and are asked your opinion of which grade level this question is appropriate for. How might you decide?

3. Explain why this function is invertible, draw the inverse in the figure above (you may change the scales along the x and y axes), and interpret the inverse.

Problem 3 (10 pts). Consider again the graph of Problems 1 and 2. Assume that the graph is that of the function

$$f(x) = \frac{80 - 20e^{\frac{10-x}{5}}}{1 + e^{\frac{10-x}{5}}}$$

- Find and carefully draw the tangent line to the graph of f at $x = 15$:



- Use this tangent line to approximate the root of the function. Then use Newton's method to find the root correct to six decimal places. How many iterations does it take to achieve this accuracy?

Problem 4 (10 pts). Consider the function

$$f(x) = \frac{1 - e^{-x}}{1 + e^{-x}}$$

1. Is it necessary to restrict f in order to define an inverse? Explain.

2. Write the formula for the inverse function g of f or its restriction.

3. Specify the domain and range of g.

Problem 5 (10 pts). Use the method of substitution to compute the integral

$$\int \left(\frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} \right) dx$$

1. as an indefinite integral, and
2. with limits of integration -5 to 5.

Show details! Correct answers without work are worth only 1 point!

Problems 6 and 7 both refer to the figure and description at the bottom of the page.
Problem 6 (10 pts). Compute the volume of wine the glass can hold.

Problem 7 (10 pts). Compute the volume of the material needed to make the glass (ignore the base, which is not shown).

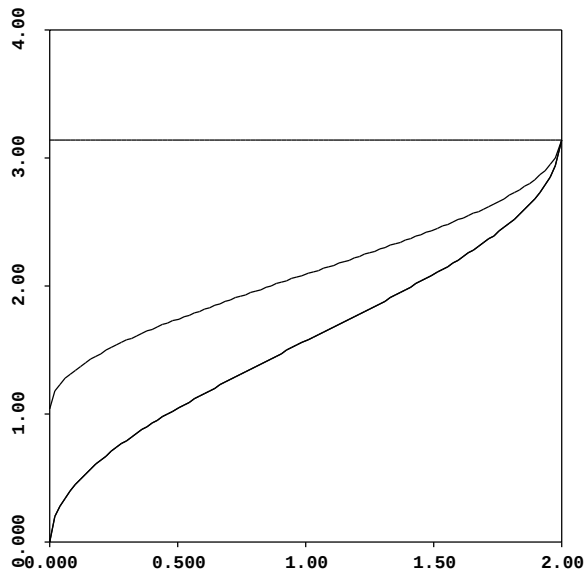
A cross section of the right half of the business portion of a wine glass (the glass is obtained by rotating this figure about the y-axis on the left edge of the figure). The base of the glass is not shown. The solid portion is bounded by graphs of the two functions

$$f(x) = \frac{\pi}{2} + \arcsin(x-1)$$

and

$$g(x) = \frac{2}{3}\left(\pi + \arcsin(x-1)\right)$$

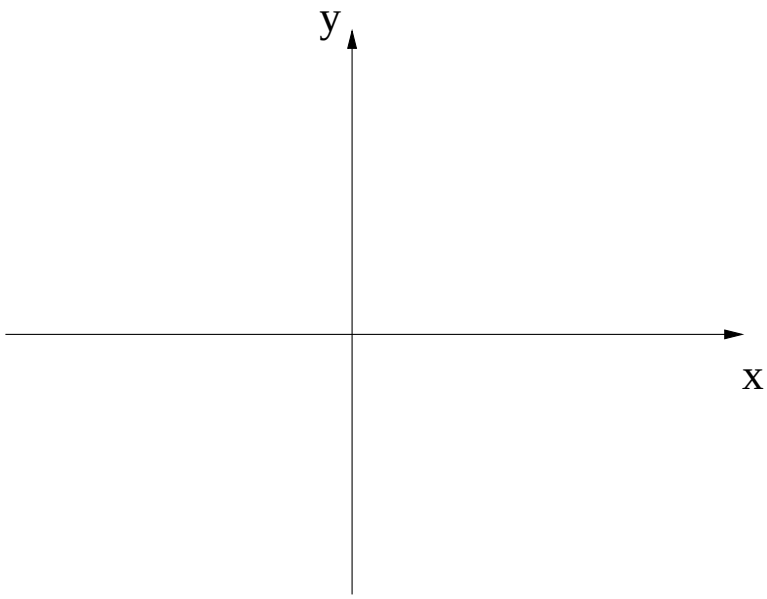
Wine can fill the glass to the level of the horizontal line $y = \pi$.



Problem 8 Study and carefully draw the graph of the function

$$f(x) = \ln\left(\frac{1}{1+x^2}\right)$$

on a suitable domain. Give details about the domain, range, symmetry, asymptotes, continuity, differentiability, max/mins, inflection points, etc. Use your calculator as needed.



Problem 9 (10 pts). Use the limit definition of the derivative to demonstrate that $\frac{d}{dx}(\frac{1}{1-x}) = \frac{1}{(1-x)^2}$.

Problem 10 (10 pts). You order a chocolate shake at Fat Farm, the Fabulous Fast Food Franchise. It comes in a container in the shape of right circular cylinder of radius 2 inches, which is filled to a height of 8 inches. If the weight density of the shake is 1 ounce per cubic inch, how much work (in foot-pounds) is required to raise the delicious liquid to your lips, which are sucking on a straw two inches above the initial surface of the shake? (For heaven's sake, draw a picture!)

Problem 11 (10 pts). Because you are a somewhat perverse calculus student, you decide to build a pen for your dog Archimedes in the shape of an isosceles triangle, with each of the two equal sides of length 100 feet.

1. What angle θ should you choose between the two equal sides so as to maximize the area of Archimedes's play space?
2. What is the length of the third side?
3. What is the resulting area?

Problem 12 (10 pts). Calculate the following *by hand!*:

1. The derivative of $\cos(\ln(x))$

2. $$\int_0^1 \ln(e^x)dx$$

3. $$\int_{-1}^1 (x^{15} + x^{13} + x^{11} + x^9 + 2x^7 - x^5 + x^3 + x + 2)dx$$