MAT120 Test 3 (Spring 2003)
Sections 4.4-5,4.7,4.10; 5.1-5.5; 6.1

Name:

Directions: Problem point values are indicated. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (20 pts) Compute the following integrals by hand (you can of course check your work with your calculator). Show your work!

$$\int_{-1}^{1} (7x^5 + 3x^3 - x^2 + x)dx$$

$$\int \cos(3x - 2)dx$$

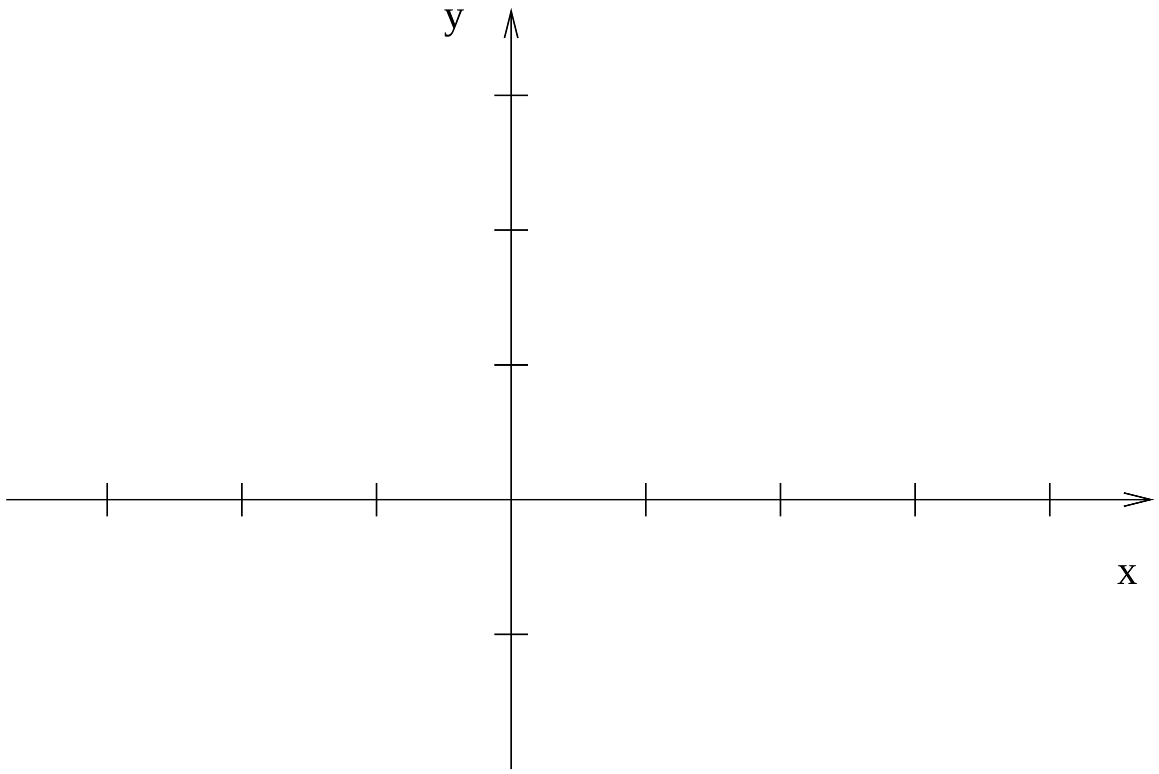
$$\int_{0}^{1} \frac{3x + 1}{(3x^2 + 2x + 2)^2}dx$$

$$\int_{-1}^{1} \frac{x}{(3x^2 - 2)^2}dx$$

Problem 2 (20 pts). For the function f given by

$$f(x) = \frac{2(x-2)}{3(x^2-4)^2}$$

study the function (resulting in a careful sketch including the important features, considering the following 7 characteristics: domain, intercepts, symmetry, asymptotes, intervals of increase and decrease, local extrema, and concavity).



Problem 3 (15 pts). A one-liter can (1000 cm^3) is to be made in the shape of a right circular cylinder.

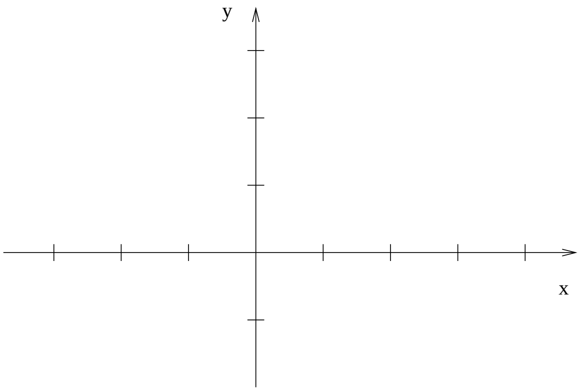
1. (10 pts) Assuming that the cost of the can is proportional to the surface area of the materials used, find the dimensions of the can which minimize the total cost.

2. (5 pts) Suppose its side and bottom will be made of cheaper materials, and the top of a more expensive material (to make it easy to open). The cost of the top material is twice that of the material used in the rest of the can. Find the dimensions that minimize total cost.

Problem 4 (20 pts). Consider the function f given by

$$f(x) = \begin{cases} 2x & 0 \le x < 2 \\ -4x + 12 & 2 \le x < 4 \\ -4 & 4 \le x \le 6 \end{cases}$$

1. Draw the graph of f on the interval [0,6].



2. On the same graph above, carefully draw the function defined by

$$g(x) = \int_0^x f(x)dx$$

evaluating it at correctly at every integer between 0 and 6.

3. Describe the relationship between g and f.

4. Using the left rectangle rule and six subintervals, write a Riemann sum that represents

$$I = \int_0^6 f(x)dx$$

and evaluate the sum.

5. Compute the exact value of the integral I above.

Problem 5 (15 pts). Find the area of the region bounded by the two curves $x = y^2$ and $y = 2x - 5$.

Problem 6 (10 pts). Consider

$$f(x) = \frac{x^3 - 2x^2 + x - 4}{x^2 - x + 1}$$

- Demonstrate that f has a slant asymptote.

- Sketch the function.