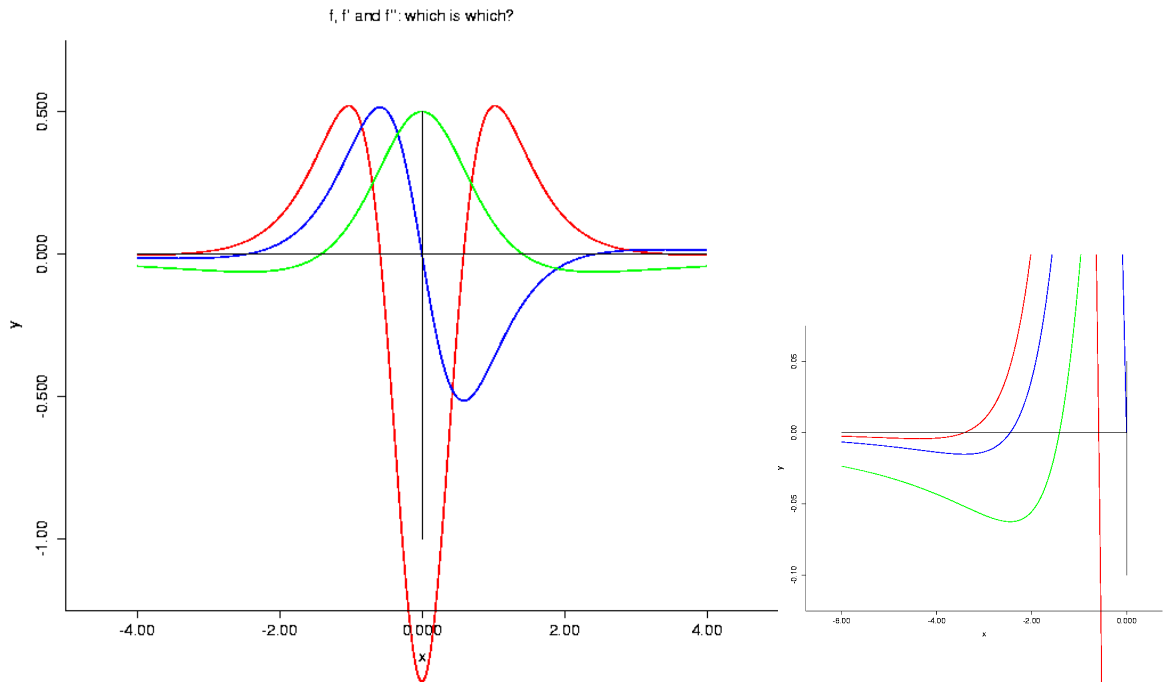
MAT121 Test 3 (Fall 2007): Sections 3.8-4.4

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. The graph below is of a function f, its derivative, and its second derivative (image at left), as well as a closeup around the x-axis on the left-hand side (right):



a. (3 pts) Identify the graph of f with an "A", f' with a "B", and f'' with a "C".

b. (7 pts) Fill in the following table as well as you can, based on what you can see above, defining critical points (from left to right) called c_1, etc., and inflection points called i_1, etc. Your table should reflect sign information, zeros, monotonicity, extrema, concavity, etc., and be entirely consistent with the function above.

x	-- >
f	
f'	
f''	

Problem 2. Consider the equation of an ellipse:

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

where a and b are parameters that determine the ellipse's eccentricity (how far it differs from a circle), and x_0 and y_0 are fixed constants.

a. (3 pts) Compute dy/dx implicitly.

b. (3 pts) Find the points on the ellipse at which the tangent is horizontal.

c. (3 pts) Solve for y as an explicit function of x to get the two branches of the function.

d. (1 pt) What condition on a and b results in the ellipse becoming a circle?

Problem 3. With a devilish grin, your two-year old child pours a half gallon (64 ounces) of orange juice onto the floor, pouring at a constant rate of 1 ounce per second, the spill growing in a circle.

The spill has thickness $\frac{1}{16}$ ounces/square inch. The volume of the spill is its thickness times its area.

a. (3 pts) At what rate is the **radius** r of the circle changing, when $r = 2$ inches?

b. (3 pts) At what rate is the **area** A of the circle changing?

c. (3 pts) At what rate is the **volume** V of the spill changing? (Think about it!)

d. (1 pt) What's the radius of the spill when the juice runs out?

Problem 4. The acceleration due to gravitational attraction between you and the earth is given by the function

$$a(x) = \frac{GM}{x^2}$$

where G is a universal constant (G = 6.673E-11 $\frac{Nm^2}{kg^2}$), M = 5.98E+24 kgs, and x is the distance between you and the center of the Earth (in meters).

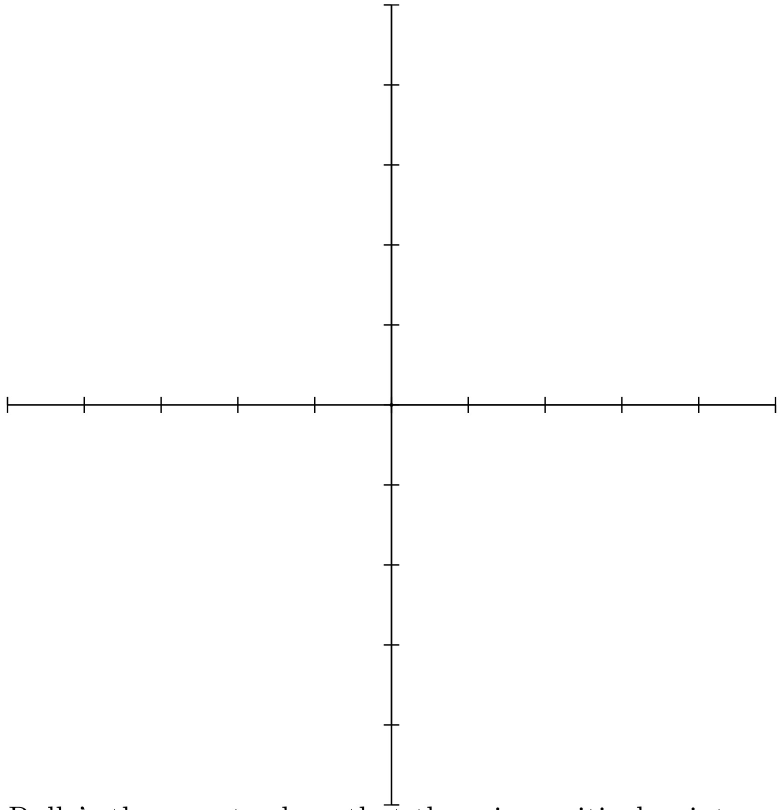
a. (1 pt) Compute the function value $a(x)$ when $x = 6,360,000m$ (which is approximately the Earth's radius). This value should be familiar.

b. (6 pts) Find the linearization $L(x)$ of the function $a(x)$ at a value of x of $6,360,000m$.

c. (2 pts) The Earth's equatorial radius is approximately 6,378,135 m, whereas its polar radius is approximately 6,356,750 m. Use the linearization to approximate the acceleration due to gravity at these extremes.

d. (1 pts) Compare the estimates of part c. to the actual values obtained using $a(x)$.

Problem 5. Sketch the graph of a function on [-5,5] such that



a. one can invoke Rolle's theorem to show that there is a critical point on the interval (3,5);

b. one can invoke the Mean Value Theorem to show that there is a point on the interval (1,3) where the derivative is -1.

c. The function is continuous on the interval [-5,5].

d. The function is not differentiable at $x = -1$.

e. An absolute maximum occurs at an endpoint, and an absolute minimum occurs at an internal critical point.