a placeholder that is used to define the rule for the derivative function.

- Given the graph of a function $y = f(x)$, we can sketch an approximate graph of its derivative $y = f'(x)$ by observing that *heights* on the derivative's graph correspond to *slopes* on the original function's graph.
- In Activity 1.4.2, we encountered some functions that had sharp corners on their graphs, such as the shifted absolute value function. At such points, the derivative fails to exist, and we say that f is not differentiable there. For now, it suffices to understand this as a consequence of the jump that must occur in the derivative function at a sharp corner on the graph of the original function.

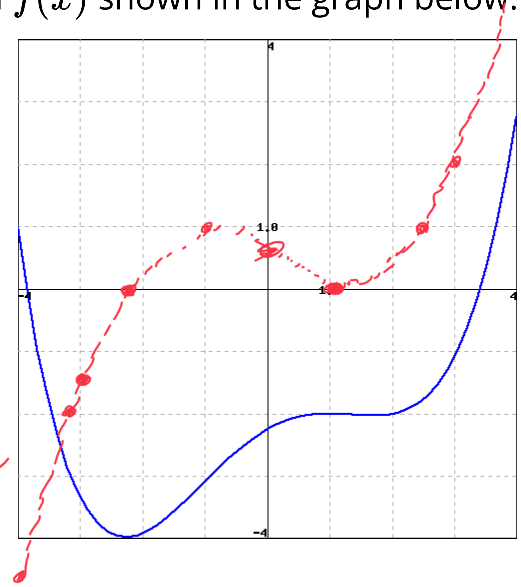
1.4.3 Exercises

1. The derivative function graphically.

Activate

Consider the function $f(x)$ shown in the graph below.

I estimated at a few obvious points (where slopes are 0, ±1, ±2, etc.) & then roughed it in.



(Note that you can click on the graph to get a larger version of it, and that it may be useful to print that larger version to be able to work with it by hand.)

Carefully sketch the derivative function of the given function (you will want to estimate values on the derivative function at different x values as you do this). Use your derivative function graph to estimate the following values on the derivative function.

at $x =$	-3	-1	1	3
the derivative is	-1.5	1	0	2

2. Applying the limit definition of the derivative.

Activate

Find a formula for the derivative of the function $g(x) = 4x^2 - 8$ using difference quotients:

$g'(x) = \lim_{h \to 0} [($ $4(x+h)^2 - 8 - (4x^2-8)$ $)/h]$

$4(x+h)^2 = 4(x^2+2xh+h^2) = 4x^2+8xh+4h^2$

$= \lim_{h\to 0}\left(\frac{8xh+4h^2}{h}\right) = \lim_{h\to 0}(8x+4h)$

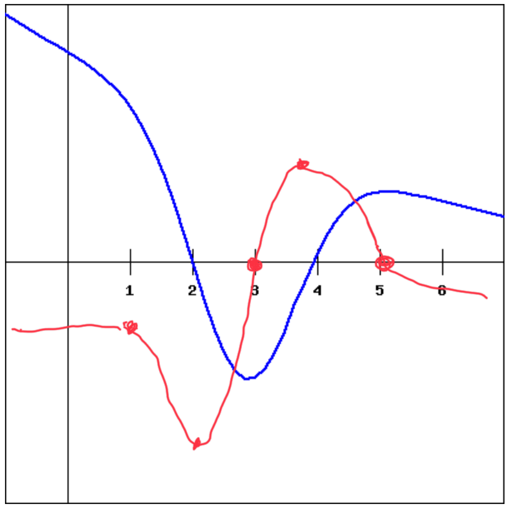
$= \boxed{8x}$

(In the first answer blank, fill in the numerator of the difference quotient you use to evaluate the derivative. In the second, fill out the derivative you obtain after completing the limit calculation.)

3. Sketching the derivative.

Activate

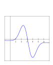
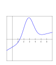
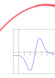
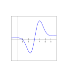
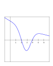
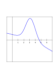
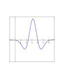
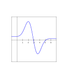
For the function $f(x)$ shown in the graph below, sketch a graph of the derivative. You will then be picking which of the following is the correct derivative graph, but should be sure to first sketch the derivative yourself.



Which of the following graphs is the derivative of $f(x)$?

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

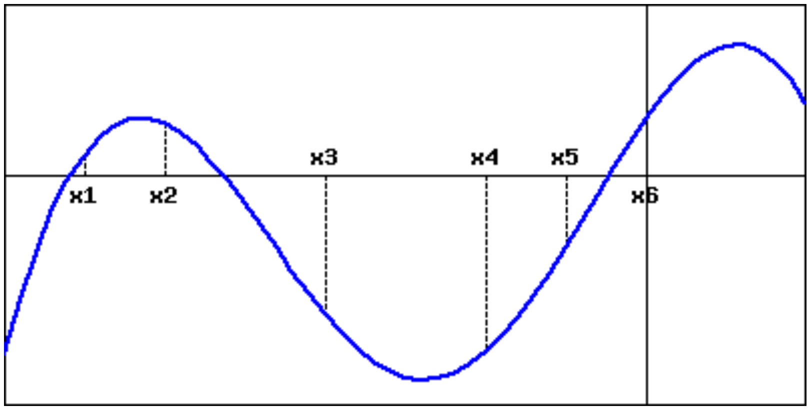
(Click on a graph to enlarge it.)

1. 	***2.*** 	***3.*** 	***4.*** 
5. 	***6.*** 	***7.*** 	***8.*** 

4. Comparing function and derivative values.

Activate

The graph of a function f is shown below.



At which of the labeled x-values is

$f(x)$ least? $x =$

- x1
- x2
- x3
- x4
- x5
- x6

$f(x)$ greatest? $x =$

- x1
- x2
- x3
- x4
- x5
- x6

$f'(x)$ least? $x =$

- x1
- x2

x3

- x3
- x4
- x5
- x6

$f'(x)$ greatest? $x =$

- x1
- x2
- x3
- x4
- x5
- x6

Tie, w/ x_1 right behind!

5. Limit definition of the derivative for a rational function.

Activate

Let

$$f(x) = \frac{1}{x-4}$$

Find

(i) $f'(3)$ -1

(ii) $f'(5)$ -1

(iii) $f'(6)$ $-1/4$

(iv) $f'(8)$ $-1/16$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)-4} - \frac{1}{x-4}}{h}$$

$$= \lim_{h\to 0} \left[\frac{1}{x+h-4} - \frac{1}{x-4}\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{x-4-(x+h-4)}{(x+h-4)(x-4)}\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{-\cancel{h}}{(x+h-4)(x-4)}\right]\frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h-4)(x-4)} = \boxed{\frac{-1}{(x-4)^2}}$$

6. Let f be a function with the following properties: f is differentiable at every value of x (that is, f has a derivative at every point), $f(-2) = 1$, and $f'(-2) = -2$, $f'(-1) = -1$, $f'(0) = 0$, $f'(1) = 1$, and $f'(2) = 2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left(\frac{1}{2}(x+h)^2 - 1\right) - \left(\frac{1}{2}x^2 - 1\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{\tfrac{1}{2}x^2} + xh + \frac{1}{2}h^2 \cancel{-1} - \cancel{\tfrac{1}{2}x^2} \cancel{+1}}{h} = \lim_{h\to 0} \frac{xh + \frac{1}{2}h^2}{h}$$

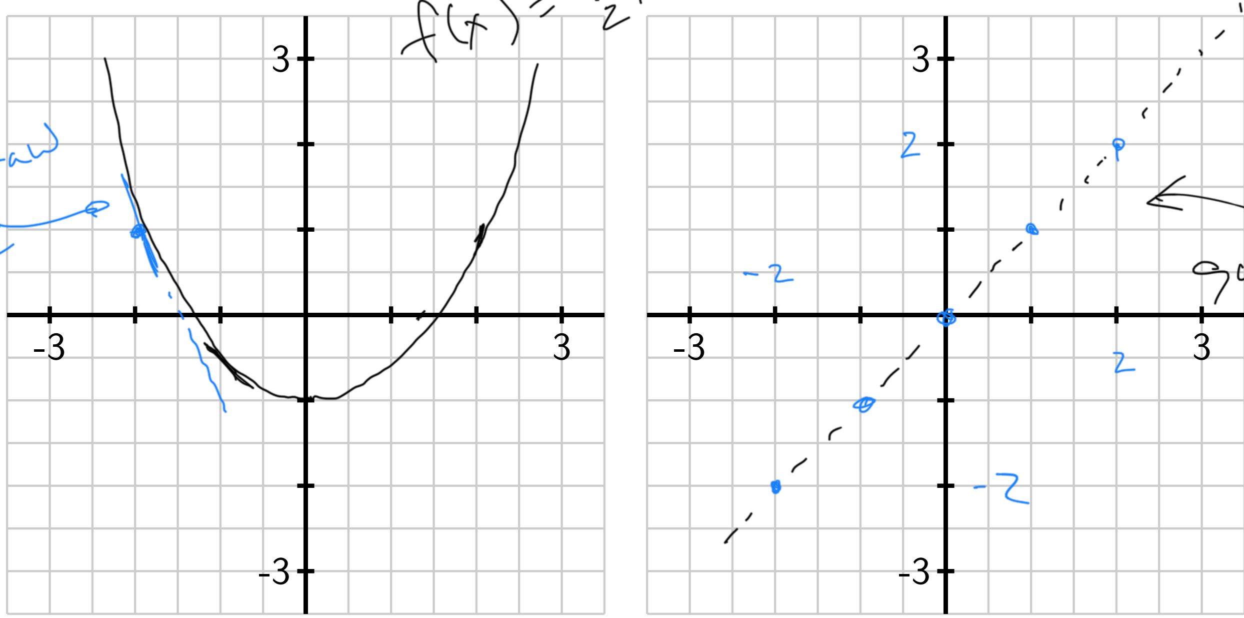
$$= \lim_{h\to 0} \frac{\cancel{h}(x+\frac{1}{2}h)}{\cancel{h}} = \lim_{h\to 0}(x+\tfrac{1}{2}h) = \boxed{x}$$

a. On the axes provided at left in Figure 1.4.3, sketch a possible graph of $y = f(x)$. Explain why your graph meets the stated criteria.

b. Conjecture a formula for the function $y = f(x)$. Use the limit definition of the derivative to determine the corresponding formula for $y = f'(x)$. Discuss both graphical and algebraic evidence for whether or not your conjecture is correct.

$f(x) = \frac{1}{2}x^2 - 1$

can draw tangent here



$f'(x) = x$

guess odd & linear, so guess f is quadratic, & even!

Figure 1.4.3. Axes for plotting $y = f(x)$ in (a) and $y = f'(x)$ in (b).

7. Consider the function $g(x) = x^2 - x + 3$.

a. Use the limit definition of the derivative to determine a formula for $g'(x)$.

b. Use a graphing utility to plot both $y = g(x)$ and your result for $y = g'(x)$; does your formula for $g'(x)$ generate the graph you expected?

c. Use the limit definition of the derivative to find a formula for $p'(x)$ where $p(x) = 5x^2 - 4x + 12$.

d. Compare and contrast the formulas for $g'(x)$ and $p'(x)$ you have found. How do the constants 5, 4, 12, and 3 affect the results?

8. Let g be a continuous function (that is, one with no jumps or holes in the graph) and suppose that a graph of $y = g'(x)$ is given by the graph on the right in Figure 1.4.4.

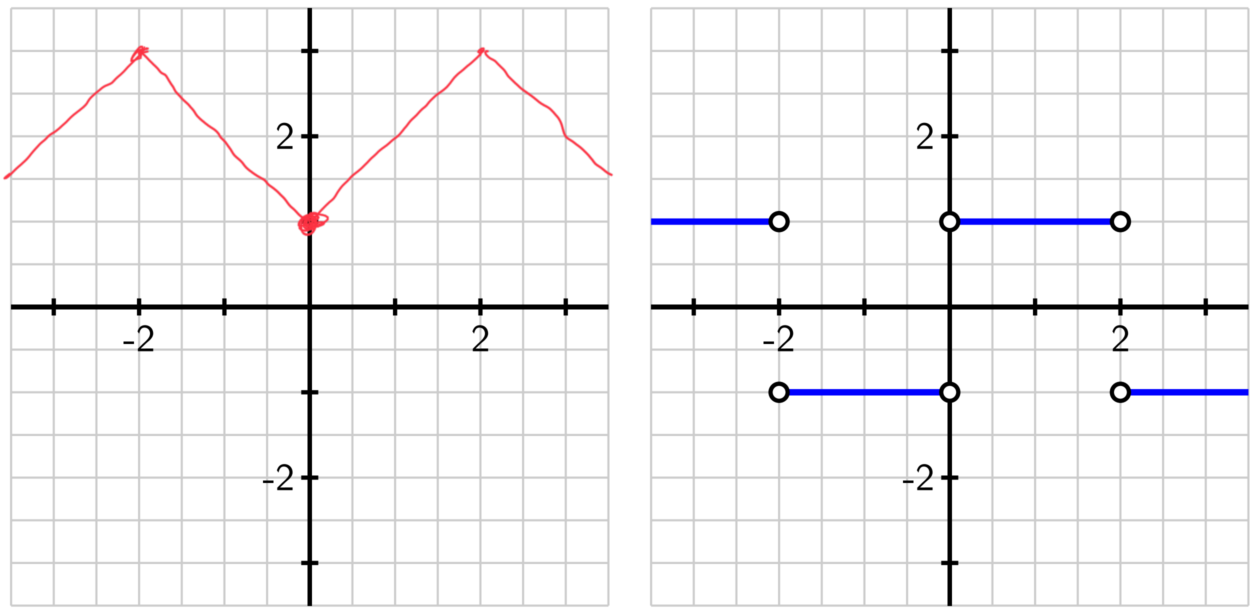


Figure 1.4.4. Axes for plotting $y = g(x)$ and, at right, the graph of $y = g'(x)$.

a. Observe that for every value of x that satisfies $0 < x < 2$, the value of $g'(x)$ is constant. What does this tell you about the behavior of the graph of $y = g(x)$ on this interval? The rate of change (instantaneous) is constant.

b. On what intervals other than $0 < x < 2$ do you expect $y = g(x)$ to be a linear function? Why? (-2,0) & (2,3.5) & (-3.5,-2) Slope constant

c. At which values of x is $g'(x)$ not defined? What behavior does this lead you to expect to see in the graph of $y = g(x)$? A corner ....

d. Suppose that $g(0) = 1$. On the axes provided at left in Figure 1.4.4, sketch an accurate graph of $y = g(x)$.

9. For each graph that provides an original function $y = f(x)$ in Figure 1.4.5, your task is to sketch an approximate graph of its derivative function, $y = f'(x)$, on the axes immediately below. View the scale of the grid for the graph of f as being 1×1, and assume the horizontal scale of the grid for the graph of f' is identical to that for f. If you need to adjust the vertical scale on the axes for the graph of f', you should label that accordingly.

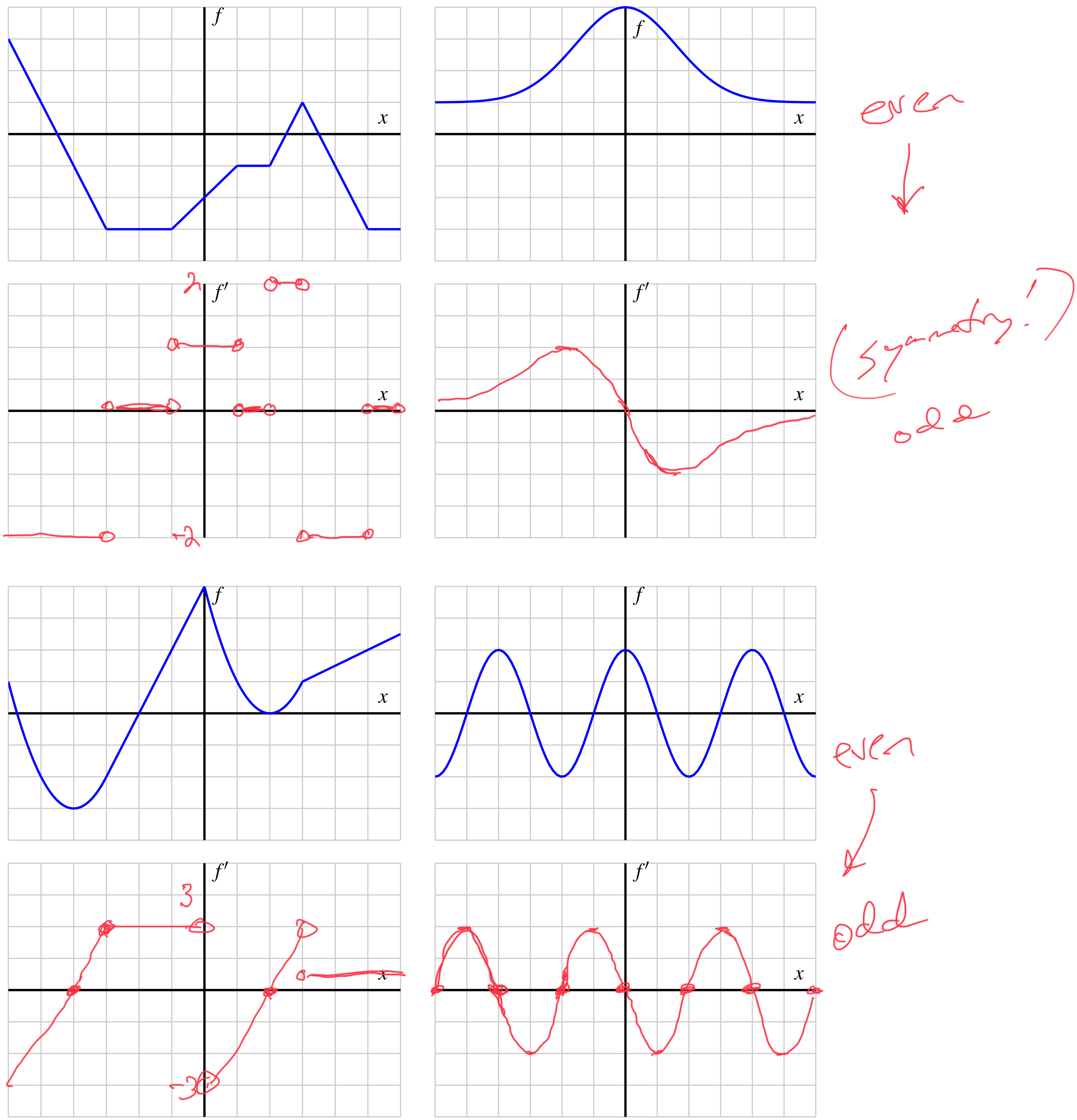


Figure 1.4.5. Graphs of $y = f(x)$ and grids for plotting the corresponding graph of $y = f'(x)$.