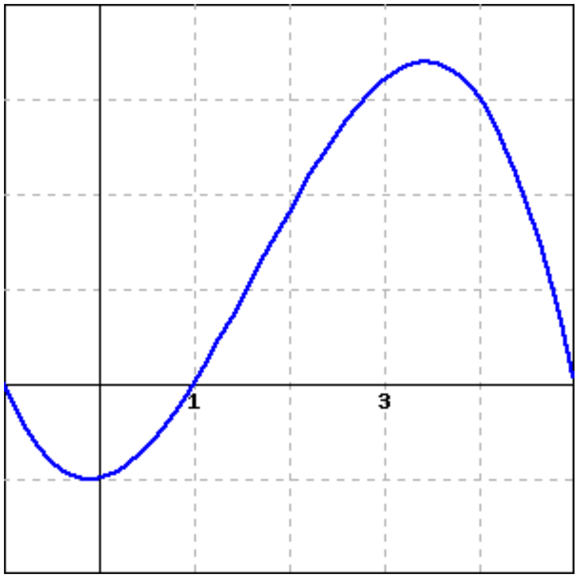
Activate

Consider the function $f(x)$ graphed below.



For this function, are the following nonzero quantities positive or negative?

$f(3)$ is

- positive
- negative

$f'(3)$ is

- positive
- negative

$f''(3)$ is

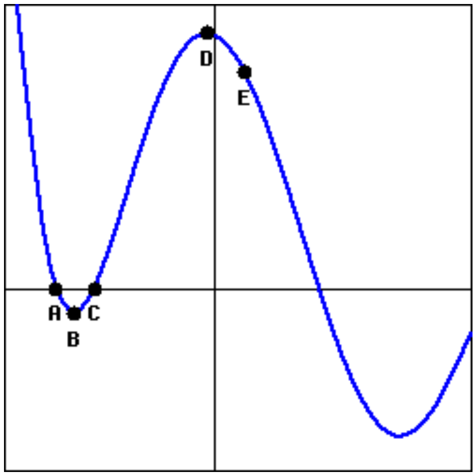
- positive
- negative

(Because this is a multiple choice problem, it will not show which parts of the problem are correct or incorrect when you submit it.)

2. Signs of f, f', f'' values.

Activate

At exactly two of the labeled points in the figure below, which shows a function f, the derivative f' is zero; the second derivative f'' is not zero at any of the labeled points. Select the correct signs for each of f, f' and f'' at each marked point.



Point	A	B	C	D	E
f	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative
f'	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative
f''	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative	▪ positive ▪ zero ▪ negative

3. Acceleration from velocity.

Activate

Suppose that an accelerating car goes from 0 mph to 64.1 mph in five seconds. Its velocity is given in the following table, converted from miles per hour to feet per second, so that all time measurements are in seconds. (Note: 1 mph is 22/15 ft/sec.) Find the average acceleration of the car over each of the first two seconds.

t (s)	0	1	2	3	4	5
$v(t)$ (ft/s)	0.00	32.05	55.55	72.64	85.45	94.00

average acceleration over the first second = 32 ft/s² help (units) [1]

$\frac{32-0}{1-0} = 32$ ft/s²

average acceleration over the second second = 22.5 ft/s² help (units) [2]

$\frac{55.55 - 32.05}{2-1} = 22.5$ ft/s²

4. Rates of change of stock values.

Activate

Let $P(t)$ represent the price of a share of stock of a corporation at time t. What does each of the following statements tell us about the signs of the first and second derivatives of $P(t)$?

(a) The price of the stock is falling slower and slower.

The first derivative of $P(t)$ is

- positive
- zero
- negative

The second derivative of $P(t)$ is

- positive
- zero
- negative

(b) The price of the stock is close to bottoming out.

The first derivative of $P(t)$ is

- positive
- zero
- negative

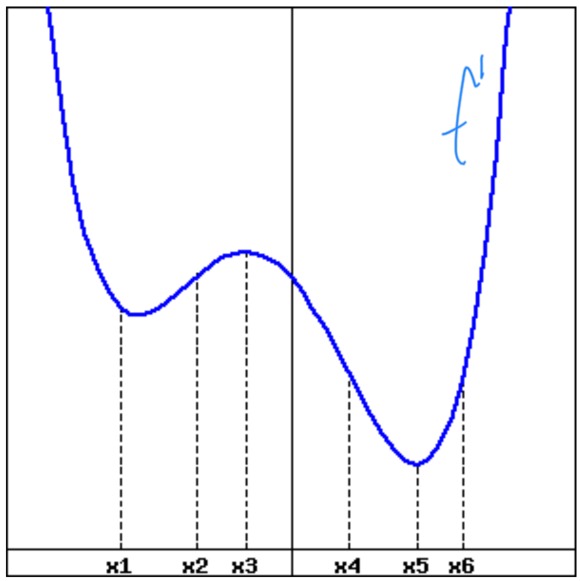
The second derivative of $P(t)$ is

- positive
- zero
- negative

5. Interpreting a graph of f'.

Activate

The graph of f' (***not*** f) is given below.



f'' is the derivative — its values can be read off the slopes of tangents.

(Note that this is a graph of f', not a graph of f.)

At which of the marked values of x is

A. $f(x)$ greatest? $x =$ x6

B. $f(x)$ least? $x =$ x1

Since $f' > 0$ everywhere, f just grows. hence smallest at left, largest at right.

C. $f'(x)$ greatest? $x =$ x3

D. $f'(x)$ least? $x =$ x5

These are just function values from the graph.

E. $f''(x)$ greatest? $x =$ x6 — slope most steeply positive.

F. $f''(x)$ least? $x =$ x4 — slope most steeply negative

6. Suppose that $y = f(x)$ is a twice-differentiable function such that f'' is continuous for which the following information is known: $f(2) = -3$, $f'(2) = 1.5$, $f''(2) = -0.25$.

The situation

a. Is f increasing or decreasing near $x = 2$? Is f concave up or concave down near $x = 2$?

$f'(2) > 0 \to$ increasing; $f''(2) < 0 \to$ concave down

b. Do you expect $f(2.1)$ to be greater than -3, equal to -3, or less than -3? Why?

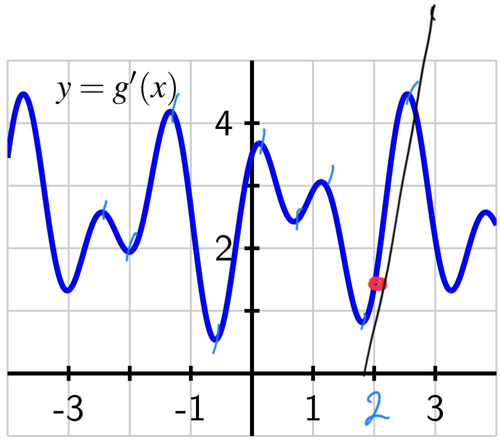
f is increasing locally

c. Do you expect $f'(2.1)$ to be greater than 1.5, equal to 1.5, or less than 1.5? Why?

Concave down → slopes are dropping.

d. Sketch a graph of $y = f(x)$ near $(2, f(2))$ and include a graph of the tangent line.

7. For a certain function $y = g(x)$, its derivative is given by the function pictured in Figure 1.6.15.



At most one, at least on the interval shown. The derivative is every- where positive → the function is increasing on the interval

Figure 1.6.15. The graph of $y = g'(x)$.

a. What is the approximate slope of the tangent line to $y = g(x)$ at the point $(2, g(2))$? About 1.3

b. How many real number solutions can there be to the equation $g(x) = 0$? Justify your conclusion fully and carefully by explaining what you know about how the graph of g must behave based on the given graph of g'.

increasing, so can cross at most once.

c. On the interval $-3 < x < 3$, how many times does the concavity of g change? Why? 9 times - 9 extrema of f'.

d. Use the provided graph to estimate the value of $g''(2)$.

Maybe m = 5 for the tangent line

8. A bungee jumper's height h (in feet) at time t (in seconds) is given in part by the table:

t	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$h(t)$	200	184.2	159.9	131.9	104.7	81.8	65.5	56.8	55.5	60.4	69.8

t	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
$h(t)$	81.6	93.7	104.4	112.6	117.7	119.4	118.2	114.8	110.0	104.7

a. Use the given data to estimate $h'(4.5)$, $h'(5)$, and $h'(5.5)$. At which of these times is the bungee jumper rising most rapidly?

b. Use the given data and your work in (a) to estimate $h''(5)$.

c. What physical property of the bungee jumper does the value of $h''(5)$ measure? What are its units?

d. Based on the data, on what approximate time intervals is the function $y = h(t)$ concave down? What is happening to the velocity of the bungee jumper on these time intervals?

9. For each prompt that follows, sketch a possible graph of a function on the interval $-3 < x < 3$ that satisfies the stated properties.

a. $y = f(x)$ such that f is increasing on $-3 < x < 3$, concave up on $-3 < x < 0$, and concave down on $0 < x < 3$.

b. $y = g(x)$ such that g is increasing on $-3 < x < 3$, concave down on $-3 < x < 0$, and concave up on $0 < x < 3$.

c. $y = h(x)$ such that h is decreasing on $-3 < x < 3$, concave up on $-3 < x < -1$, neither concave up nor concave down on $-1 < x < 1$, and concave down on $1 < x < 3$.

d. $y = p(x)$ such that p is decreasing and concave down on $-3 < x < 0$ and is increasing and concave down on $0 < x < 3$.