1. Approximating $\sqrt{x}$.

Activate

$f'(x) = \frac{1}{2\sqrt{x}}$

Use linear approximation to approximate $\sqrt{36.1}$ as follows.

$f'(36) = \frac{1}{2\sqrt{36}} = \frac{1}{12}$

Let $f(x) = \sqrt{x}$. The equation of the tangent line to $f(x)$ at $x = 36$ can be written in the form $y = mx + b$. Compute m and b.

$f(36) = 6$

$m =$ $1/12$

but why would you?

$b =$ 3

Using this find the approximation for $\sqrt{36.1}$.

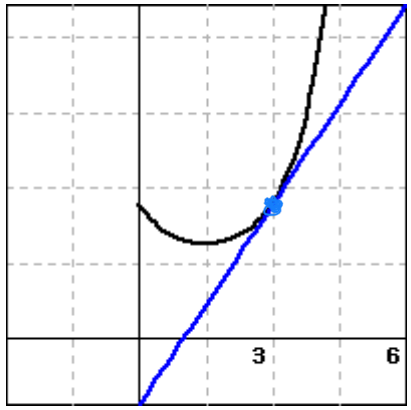
$$y = f(36) + f'(36)(x-36)$$
$$= 6 + \frac{1}{12}(x-36)$$
$$= 6 + \frac{1}{12}x - 3$$
$$y = 3 + \frac{1}{12}x$$
$$f(36.1) \approx 3 + \frac{1}{12}(36.1)$$
$$= 6 + \frac{1}{120} = 6.008\overline{3}$$

Answer: $6.008\overline{3}$

2. Local linearization of a graph.

Activate

The figure below shows $f(x)$ and its local linearization at $x = a$, $y = 4x - 4$. (The local linearization is shown in blue.)



What is the value of a?

$a =$ 3

What is the value of $f(a)$?

$f(a) =$ 8

Use the linearization to approximate the value of $f(3.2)$.

$f(3.2) =$ $4 \cdot 3.2 - 4 = 8 + 0.8 = \boxed{8.8}$

Is the approximation an under- or overestimate?

under (Enter ***under*** or ***over***.)

3. Estimating with the local linearization.

Activate

Suppose that $f(x)$ is a function with $f(130) = 46$ and $f'(130) = 1$. Estimate $f(125.5)$.

$f(125.5) \approx$ 41.5

$L(x) = 46 + f'(130)(x - 130)$
$= 46 + 1(x - 130)$
$L(125.5) = 46 + 1 \cdot (125.5 - 130)$
$= 46 - 4.5$
$= 41.5$

4. Predicting behavior from the local linearization.

Activate

The temperature, H, in degrees Celsius, of a cup of coffee placed on the kitchen counter is given by $H = f(t)$, where t is in minutes since the coffee was put on the counter.

(a) Is $f'(t)$ positive or negative?

- positive
- negative

Temperature $f(t)$ is decreasing, so $f'(t) < 0$.

(Be sure that you are able to give a reason for your answer.)

(b) What are the units of $f'(30)$? $^\circ C / min$ help (units) 4

Suppose that $|f'(30)| = 0.9$ and $f(30) = 51$. Fill in the blanks (including units where needed) and select the appropriate terms to complete the following statement about the temperature of the coffee in this case.

At 30 minutes after the coffee was put on the counter, its

- derivative
- temperature
- change in temperature

is $51^\circ C$ and will

- increase
- decrease

by about $.9\ ^\circ C / min$ in the next 75 seconds.

Note: If you are using MathQuill click the textbox (Tt) button before entering an answer that contains units.

5. A certain function $y = p(x)$ has its local linearization at $a = 3$ given by $L(x) = -2x + 5$.

a. What are the values of $p(3)$ and $p'(3)$? Why?

$p(3) = -1 \ (= -2 \cdot 3 + 5)$

$p'(3) = -2$ (the slope of the line)

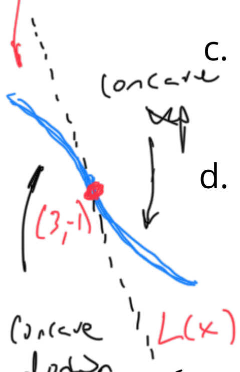
under I would write it as $L(x) = -1 + (-2)(x-3)$

estimate

b. Estimate the value of $p(2.79)$. $\simeq L(2.79) = -1 + (-2)(2.79-3) = -1 - 2(-.21) = -.58$

c. Suppose that $p''(3) = 0$ and you know that $p''(x) < 0$ for $x < 3$. Is your estimate in (b) too large or too small?

d. Suppose that $p''(x) > 0$ for $x > 3$. Use this fact and the additional information above to sketch an accurate graph of $y = p(x)$ near $x = 3$. Include a sketch of $y = L(x)$ in your work.



6. A potato is placed in an oven, and the potato's temperature F (in degrees Fahrenheit) at various points in time is taken and recorded in the following table. Time t is measured in minutes.

Table 1.8.7. *Temperature data for the potato.*

t	$F(t)$
0	70
15	180.5
30	251
45	296
60	324.5
75	342.8
90	354.5

a. Use a central difference to estimate $F'(60)$. Use this estimate as needed in subsequent questions.

b. Find the local linearization $y = L(t)$ to the function $y = F(t)$ at the point where $a = 60$.

c. Determine an estimate for $F(63)$ by employing the local linearization.

d. Do you think your estimate in (c) is too large or too small? Why?

7. An object moving along a straight line path has a differentiable position function $y = s(t)$; $s(t)$ measures the object's position relative to the origin at time t. It is known that at time $t = 9$ seconds, the object's position is $s(9) = 4$ feet (i.e., 4 feet to the right of the origin). Furthermore, the object's

$s'(9) = -1.2$ ft/s

$s''(9) = 0.08$ ft/s^2

$L(t) = s(9) + s'(9)(t-9)$

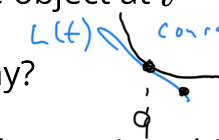
$= 4 + (-1.2)(t-9)$

$L(9.34) = 4 + (-1.2)(.34) = \boxed{3.592 \text{ ft}}$

instantaneous velocity at $t = 9$ is -1.2 feet per second, and its acceleration at the same instant is 0.08 feet per second per second.

a. Use local linearity to estimate the position of the object at $t = 9.34$. ≈ 3,592 ft.

b. Is your estimate likely too large or too small? Why?



concave up

underestimate

c. In everyday language, describe the behavior of the moving object at $t = 9$. Is it moving toward the origin or away from it? Is its velocity increasing or decreasing?

It's velocity is increasing; it is less negative.

8. For a certain function f, its derivative is known to be $f'(x) = (x-1)e^{-x^2}$. Note that you do not know a formula for $y = f(x)$.

a. At what x-value(s) is $f'(x) = 0$? Justify your answer algebraically, but include a graph of f' to support your conclusion.

b. Reasoning graphically, for what intervals of x-values is $f''(x) > 0$? What does this tell you about the behavior of the original function f? Explain.

c. Assuming that $f(2) = -3$, estimate the value of $f(1.88)$ by finding and using the tangent line approximation to f at $x = 2$. Is your estimate larger or smaller than the true value of $f(1.88)$? Justify your answer.