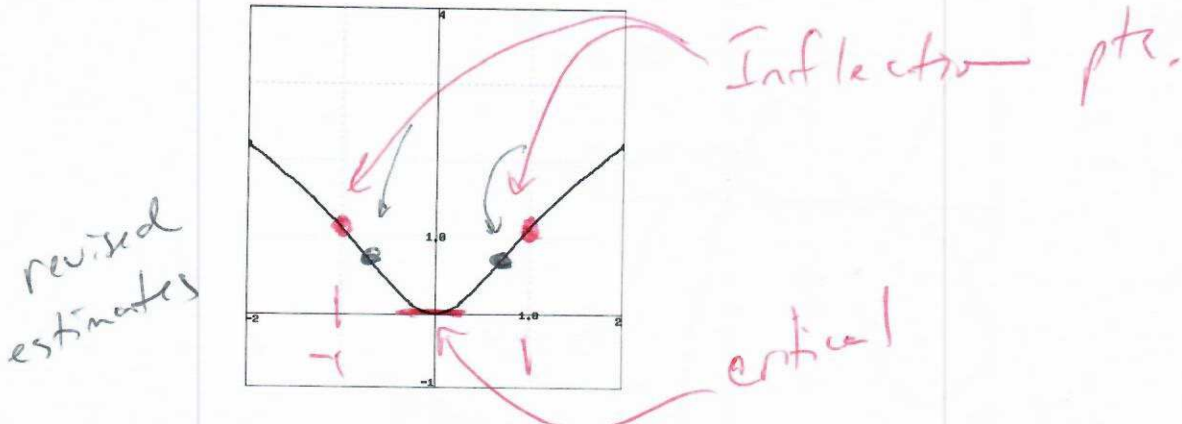
Use a graph below of $f(x) = \ln(2x^2 + 1)$ to estimate the x-values of any critical points and inflection points of $f(x)$.



critical points (enter as a comma-separated list): $x =$ 0

inflection points (enter as a comma-separated list): $x =$ -1, 1

$$f'(x) = \frac{1}{2x^2+1} \cdot (2x^2+1)' = \frac{4x}{2x^2+1} = 0 \text{ when } \boxed{x=0}$$

Next, use derivatives to find the x-values of any critical points and inflection points exactly.

critical points (enter as a comma-separated list): $x =$ 0

inflection points (enter as a comma-separated list): $x =$ $-\sqrt{2}/2$, $\sqrt{2}/2$

$$f''(x) = \frac{4(2x^2+1) - (4x)^2}{(2x^2+1)^2} = \frac{8x^2+4-16x^2}{(2x^2+1)^2} = \frac{4(1-2x^2)}{(2x^2+1)^2}$$

2. Finding inflection points.

Activate

Find the inflection points of $f(x) = 4x^4 + 55x^3 - 21x^2 + 3$. (Give your answers as a comma separated list, e.g., **3,-2**.)

inflection points = -7, 1/8

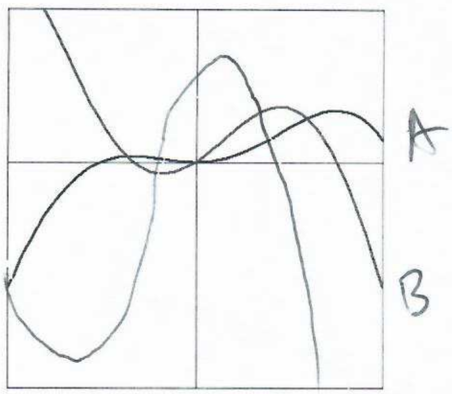
$$f'(x) = 16x^3 + 165x^2 - 42x$$

$$f''(x) = 48x^2 + 330x - 42 = 6[8x^2 + 55x - 7] = 0$$

when $x = -7$ or $x = \frac{1}{8}$

3. Matching graphs of f, f', f''.

The following shows graphs of three functions, A (in black), B (in blue), and C (in green). If these are the graphs of three functions f, f', and f'', identify which is which.



(Click on the graph to get a larger version.)

C printer didn't print the green

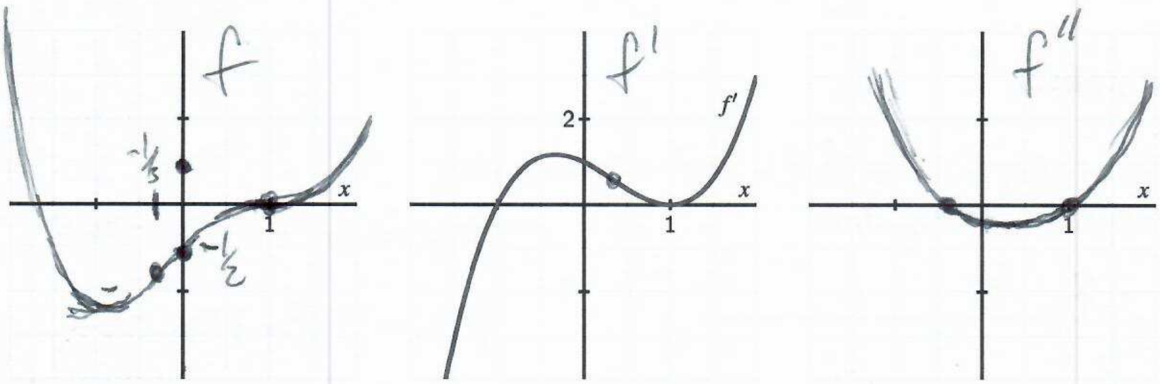
(For each enter A, B or C).

$f =$ A; $f' =$ B; $f'' =$ C

Locations of zeros suffices.

4. This problem concerns a function about which the following information is known:

- f is a differentiable function defined at every real number x
- $f(0) = -1/2$
- $y = f'(x)$ has its graph given at center in Figure 3.1.17



f''(x)

Figure 3.1.17. At center, a graph of $y = f'(x)$; at left, axes for plotting $y = f(x)$; at right, axes for plotting $y = f''(x)$.

a. Construct a first derivative sign chart for f. Clearly identify all critical numbers of f, where f is increasing and decreasing, and where f has local extrema.

b. On the right-hand axes, sketch an approximate graph of $y = f''(x)$.

c. Construct a second derivative sign chart for f. Clearly identify where f is concave up and concave down, as well as all inflection points.

Feedback

x			-1		$-\frac{1}{3}$		0		1		
$f''(x)$	∞	$+$			0	$-$			0	$+$	∞
$f'(x)$	$-\infty$	$-$	0	$+$					0	$+$	∞
$f(x)$	∞	↘	$-$	↗			$-\frac{1}{2}$	↗	$-$	↗	∞

f' looks cubic $\rightarrow$ f'' should look quadratic

f should look quartic

5. Suppose that g is a differentiable function and $g'(2) = 0$. In addition, suppose that on $1 < x < 2$ and $2 < x < 3$ it is known that $g'(x)$ is positive.

a. Does g have a local maximum, local minimum, or neither at $x = 2$? Why? inflection!

b. Suppose that $g''(x)$ exists for every x such that $1 < x < 3$. Reasoning graphically, describe the behavior of $g''(x)$ for x-values near 2.

$1 < x < 2$ negative
$2 < x < 3$ positive

c. Besides being a critical number of g, what is special about the value $x = 2$ in terms of the behavior of the graph of g?

It's a point of inflection.

6. Suppose that h is a differentiable function whose first derivative is given by the graph in Figure 3.1.18.

a. How many real number solutions can the equation $h(x) = 0$ have? Why? At most two! $x \geq 0$

b. If $h(x) = 0$ has two distinct real solutions, what can you say about the signs of the two solutions? Why? One positive, one negative — They're on opposite sides of $x = 0$!

c. Assume that $\lim_{x\to\infty} h'(x) = 3$, as appears to be indicated in Figure 3.1.18. How will the graph of $y = h(x)$ appear as $x \to \infty$? Why?

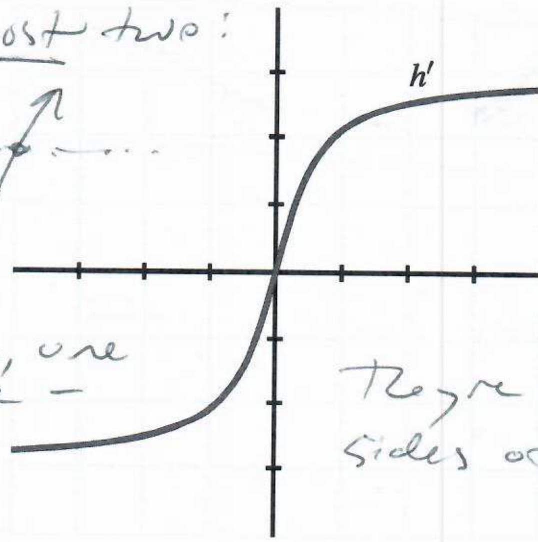


Figure 3.1.18. The graph of $y = h'(x)$.

It will have constant slope of ~3 – so like a line w/ slope 3.

d. Describe the concavity of $y = h(x)$ as fully as you can from the provided information.

Second derivative > 0 everywhere → concave up everywhere

7. Let p be a function whose second derivative is $p''(x) = (x + 1)(x - 2)e^{-x}$.

a. Construct a second derivative sign chart for p and determine all inflection points of p.

b. Suppose you also know that $x = \frac{\sqrt{5}-1}{2}$ is a critical number of p. Does p have a local minimum, local maximum, or neither at $x = \frac{\sqrt{5}-1}{2}$? Why?

c. If the point $(2, \frac{12}{e^2})$ lies on the graph of $y = p(x)$ and $p'(2) = -\frac{5}{e^2}$, find the equation of the tangent line to $y = p(x)$ at the point where $x = 2$. Does the tangent line lie above the curve, below the curve, or neither at this value? Why?