Let $F(x) = f(g(x))$. Suppose $g'(x) \neq 0$.

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$\approx \lim_{h \to 0} \frac{f(g(x) + hg'(x)) - f(g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(g(x) + hg'(x)) - f(g(x))}{h} \cdot \frac{g'(x)}{g'(x)}$$

$$= \lim_{h \to 0} \frac{f(g(x) + hg'(x)) - f(g(x))}{\underbrace{hg'(x)}} \cdot g'(x)$$

Let $hg'(x) = k$ (change of variables)

$$= \underbrace{\lim_{k \to 0} \frac{f(g(x) + k) - f(g(x))}{k}} \cdot g'(x)$$

$$= f'(g(x)) \cdot g'(x)$$

Let $F(x) = \sin(x)$

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\left[\cos(h) - 1\right] + \sin(h)\cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\left[\cos(h) - 1\right]}{h} + \lim_{h \to 0} \frac{\sin(h)\cos(x)}{h}$$

$$= \sin(x) \lim_{h \to 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$$

$$= \sin(x) \underbrace{\lim_{h \to 0} \frac{\cos(h) - \cos(0)}{h}}_{= \cos'(0) = 0} + \cos(x) \underbrace{\lim_{h \to 0} \frac{\sin(x) - \overset{0}{\overset{\parallel}{\sin(0)}}}{h}}_{\sin'(0) = 1}$$

$$= \cos(x)$$

Let $f(x) = \dfrac{\sin(x)}{\cos(x)} = \sin(x) \cdot \dfrac{1}{\cos(x)}$

$$f'(x) = \frac{\cos(x)\cos(x) - \sin(x)\left(-\sin(x)\right)}{\cos(x)^2}$$

$$= \frac{\cos(x)^2 + \sin(x)^2}{\cos(x)^2}$$

$$= \frac{1}{\cos(x)^2}$$

Pythagorean Identity

$$= \left(\frac{1}{\cos(x)}\right)^2$$

$$= \sec(x)^2$$

$$f'(x) = \left(\sin(x) \cdot \frac{1}{\cos(x)}\right)'$$

$\sec'(x)$

$$= \cos(x) \cdot \frac{1}{\cos(x)} + \sin(x)\left(\frac{1}{\cos(x)}\right)'$$

$$= 1 + \sin(x) \cdot \frac{-(-\sin(x))}{(\cos(x))^2}$$

$$= 1 + \frac{\sin(x)^2}{\cos(x)^2}$$

$$= 1 + \tan(x)^2 \quad \left(= \sec(x)^2 \right)$$

$$\boxed{\tan'(x) = 1 + \tan(x)^2}$$

$$\sec'(x) = \frac{\sin(x)}{\cos(x)^2} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)}$$

$$= \tan(x) \cdot \sec(x)$$

Let $f(x) = \sqrt{x}$ Want local linearization about $x = 1$
($a = 1$)

$f(x) = x^{1/2}$ Domain: $[0, \infty)$

$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}}$

$= \frac{1}{2\sqrt{x}}$

$L(x) = f(a) + f'(a)(x - a)$

$= f(1) + f'(1)(x - 1)$

$= 1 + \frac{1}{2}(x - 1)$

$f(1.3) \approx L(1.3) = 1 + \frac{1}{2}(1.3 - 1)$

(1.140) $= 1 + \frac{1}{2}(0.3) = 1.15$

$f(0.7) \approx L(0.7) = 1 + \frac{1}{2}(-0.3) = 0.85$

(0.894)

#4 $h(x) = \dfrac{\sin(x)}{1+x^2}$ Domain : $\mathbb{R}$

$$h'(x) = \left(\sin(x) \cdot \frac{1}{1+x^2}\right)'$$

$$= \cos(x) \cdot \frac{1}{1+x^2} + \sin(x) \frac{-2x}{(1+x^2)^2}$$

$$= \frac{\cos(x)(1+x^2) - 2x\sin(x)}{(1+x^2)^2}$$

Want the local linearization about $a=0$.

$h(0) = 0 \quad h'(0) = 1$

$L(x) = h(0) + h'(0)(x-0) = x$