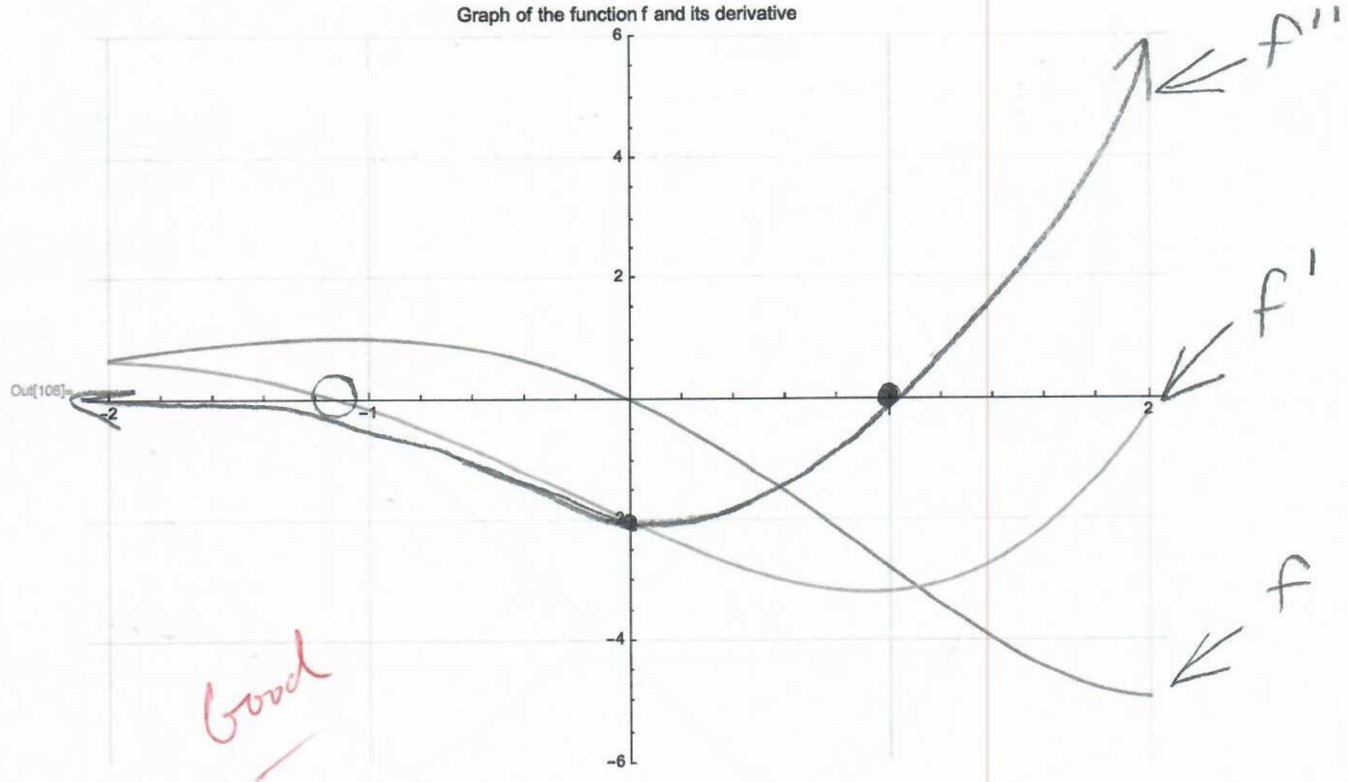
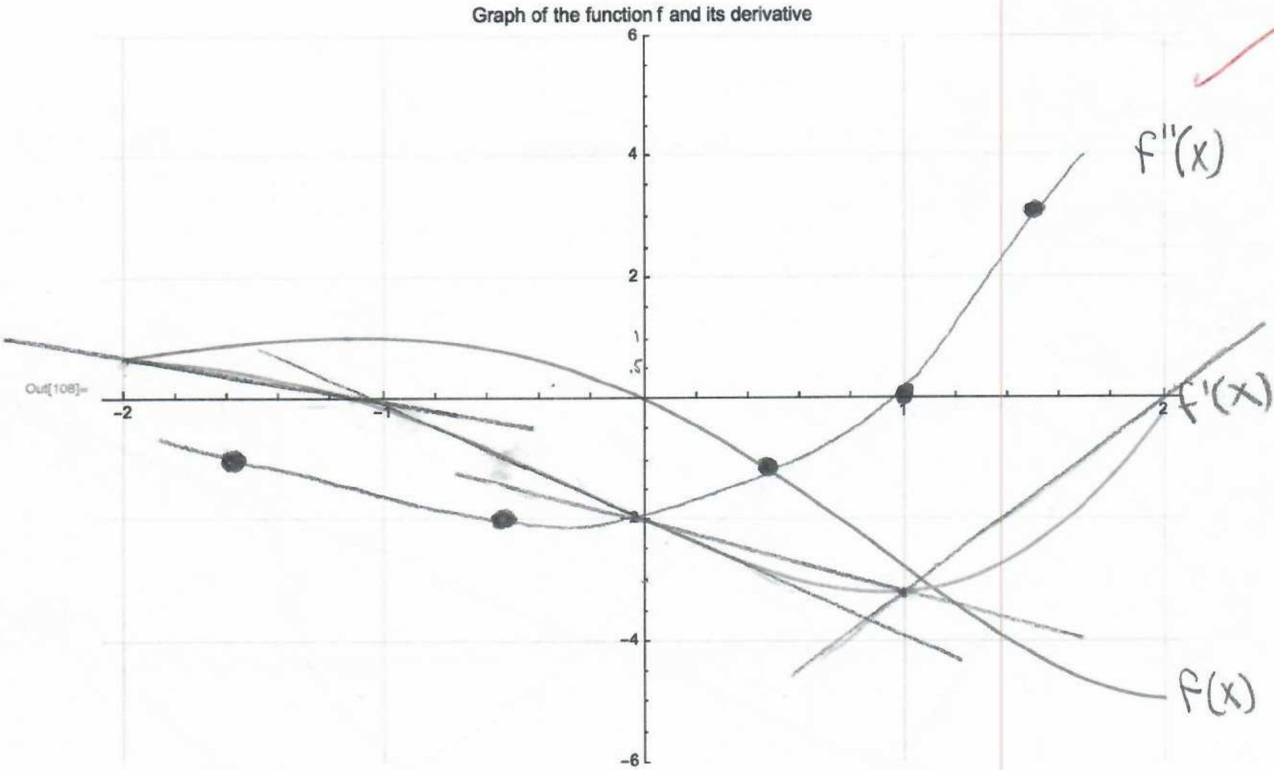
1. (4 pts) Below is a plot of f and f' (which is which? Label them appropriately). Carefully add the second derivative function f''(x) to the graph, using estimates from slopes of tangent lines.



Relate zeros of the derivative and second derivative to features of f.

The zero of f' circled at around -1 correlates to the peak of the f graph's arc where the slope is zero.
The darkly marked f'' zero correlates to the concavity change of f, & the minimum of f'.

1. (4 pts) Below is a plot of f and f' (which is which? Label them appropriately). Carefully add the second derivative function f''(x) to the graph, using estimates from slopes of tangent lines.



Relate zeros of the derivative and second derivative to features of f.

The zero of the f'(x) function would come from the local extrema of the f(x) function.

The zero of the f''(x) function would come from the local extrema of the f'(x) function.

good

2. (a) (3 pts) Consider the function $\boxed{f(x) = 1 - 4x + x^3}$. Its derivative is $\boxed{f'(x) = -4 + 3x^2}$. Below you see their graphs. Use the limit definition to find an algebraic expression for the second derivative function f''(x).

$$f''(x) = \lim_{h\to 0} \frac{(-4+3(x+h)^2) - (-4+3x^2)}{h}$$

$$= \lim_{h\to 0} \frac{(-4+3(x^2+2xh+h^2)) - (-4+3x^2)}{h}$$

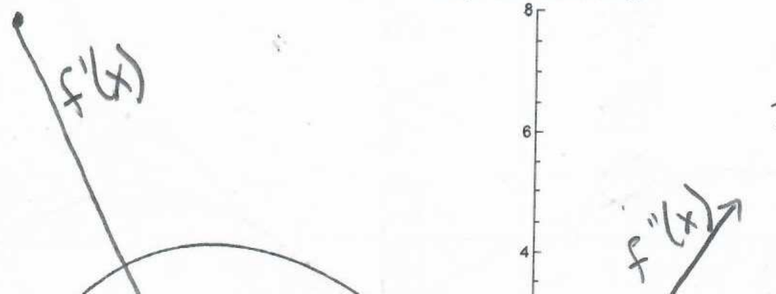
$$= \lim_{h\to 0} \left(\frac{\cancel{-4}+\cancel{3x^2}+6xh+3h^2-\cancel{(-4+3x^2)}}{h} \right)$$

$$= \lim_{h\to 0} \left(\frac{6xh+3h^2}{h} \right)$$

$$= \lim_{h\to 0} \left(\frac{\cancel{h}(6x+3h)}{\cancel{h}} \right)$$

$$\boxed{f''(x) = 6x}$$

Graph of the function f(x)



$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = x^3 - 4x + 1$

$f'(x) = 3x^2 - 4$

$f''(x) = 6x$

$(x+h)(x+h)$

$x^2 + hx + hx + h^2$

$3 \cdot (x^2 + 2hx + h^2)$

2. (a) (3 pts) Consider the function $f(x) = 1 - 4x + x^3$. Its derivative is $f'(x) = -4 + 3x^2$. Below you see their graphs. Use the limit definition to find an algebraic expression for the second derivative function f''(x).

$f''(x) = \lim_{h\to 0} \frac{3(x+h)^2 - 4 - (3x^2 - 4)}{h}$

$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 \cancel{-4} - \cancel{3x^2} + \cancel{4}}{h}$

$= \lim_{h\to 0} \frac{6hx + 3h^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$

$= \lim_{h\to 0} (6x + 3h)$

$\to 6x + 3(0)$

$= 6x$

$f''(x) = 6x$

Nice work

$x^3 - 4x + 1$ $\qquad\qquad\qquad\qquad$ $3x^2 - 4$

2. (a) (3 pts) Consider the function $f(x) = 1 - 4x + x^3$. Its derivative is $f'(x) = -4 + 3x^2$. Below you see their graphs. Use the limit definition to find an algebraic expression for the second derivative function f''(x).

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h)^2 - 4] - [3x^2 - 4]}{h}$$

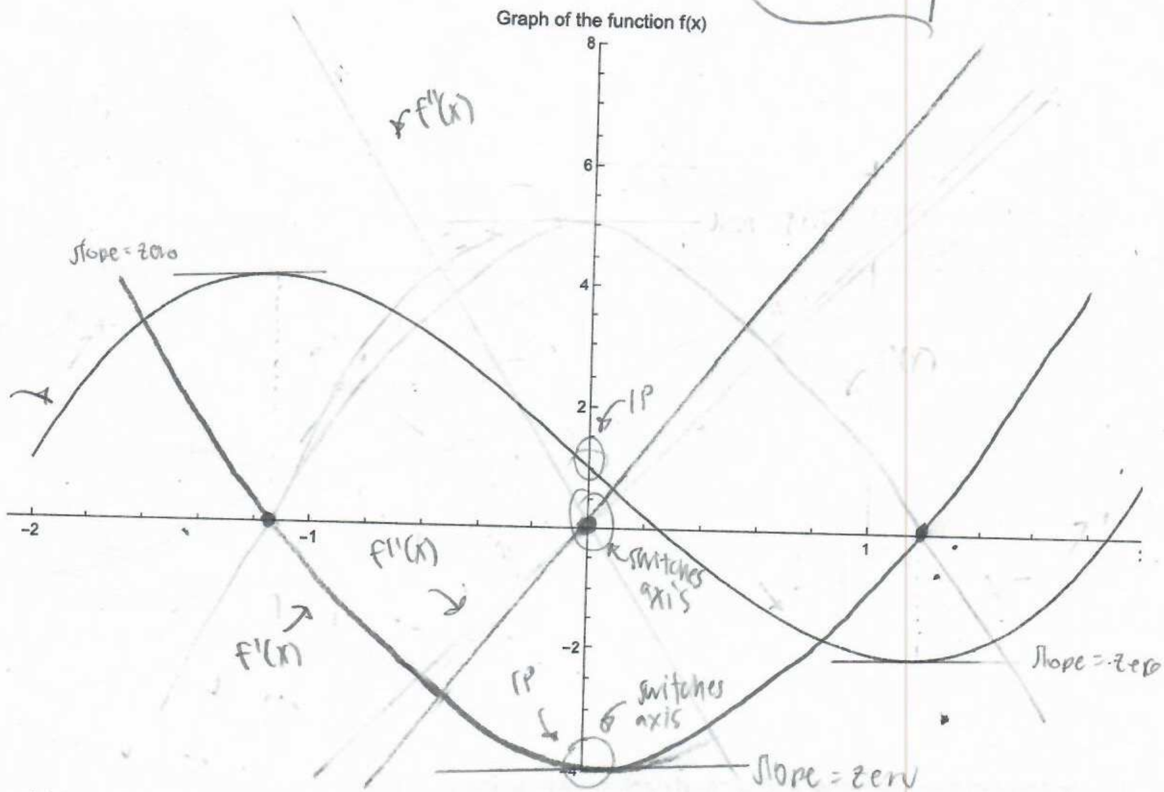
$$= \lim_{h \to 0} \frac{[3(x^2 + 2xh + h^2) - 4] - [3x^2 - 4]}{h}$$

$$= \lim_{h \to 0} \frac{(3x^2 + 6xh + 3h^2 - 4) - (3x^2 - 4)}{h}$$

$$= \lim_{h \to 0} \frac{6x\cancel{h} + 3h^2}{\cancel{h}}$$

$$= \lim_{h \to 0} (6x + 3\cancel{h}^{\to 0})$$

$$\boxed{f''(x) = 6x}$$



Nice

2. (b) (3 pts) Add the graphs of f'(x) and f''(x) onto the graph above, and explain why the derivatives make sense -- how do they relate to the behavior of f?

The derivatives make sense because each derivate decreases the exponent value (from $x^3 \to x^2 \to x \to a$). For example, $f(x) = 3x^3 - 4x + 1$ and the derivative is $3x^2 - 4$, so therefore the next derivative must be linear ($6x$ in this case) by following the $x^3 \to x^2 \to x \to a$ pattern.