Section 2.1 and 2.3 Worksheet

Power and Linear Rules Worksheet

1. Let $f(x) = 2x^3 - 15x^2 + 24x - 10$.

1.1. Compute the derivative $f'(x)$.

$(2x^3 - 15x^2 + 24x - 10)' = f'(x)$

$(2x^3)' - (15x^2)' + (24x) - (10)' = 2(x^3)' - 15(x^2)' + 24(x)' - 0$

$= 2 \cdot 3x^{3-1} - 15 \cdot 2x^{2-1} + 24x^{1-1}$

$f'(x) = 6x^2 - 30x + 24$

1.2. What is the slope of the tangent line when $x = 0$?

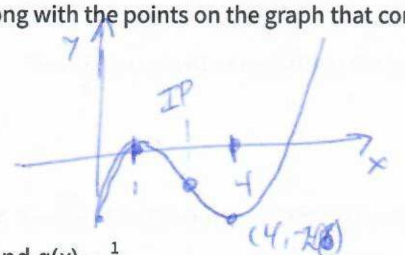
$f'(0) = 24$

1.3. What are the x-values for points on $y = f(x)$ where the slope of the tangent line is 0? (This is NOT the same question as 2.2. That was to find $f'(0)$, while this is to solve $f'(x) = 0$ for x.)

Slope of the tangent line $= 0 \rightarrow f'(x) = 0$

$f'(x) = 6(x^2 - 5x + 4) = 6(x-1)(x-4) = 0$ when $x = 1$ or $x = 4$

1.4. Using something like Desmos or a graphing calculator, graph $y = f(x)$ for $-1 \le x \le 6$. Sketch the results below along with the points on the graph that correspond to the x-values you got in 2.3.



$f(4) = ~~128~~ - 240 + 96 - 10$

$= ~~128~~ - 26$

$f(1) =$

2. Let $f(x) = x^5$ and $g(x) = \frac{1}{x^5}$.

2.1. What is $f'(x)$ and what is $g'(x)$?

$f'(x) = (x^5)' = 5x^4$; $g'(x) = \left(\frac{1}{x^5}\right)' = (x^{-5})' = -5x^{-5-1}$

$= -5x^{-6}$

2.2. What are the fourth derivatives of each, $f^{(4)}(x)$ and $g^{(4)}(x)$?

$f^{(4)}(x) = (f^{(3)}(x))' = ((f''(x))')' = (((f'(x))')')'$

$= (((5x^4)')')' = ((5 \cdot 4 \cdot x^3)')' = (5 \cdot 4 \cdot 3 \cdot x^2)' = 5 \cdot 4 \cdot 3 \cdot 2 \cdot x$

$g^{(4)}(x) =$

$(-5)(-6)(-7)(-8)x^{-9}$

These "factorials" tend to develop.

2.3. What are the sixth derivatives of each, $f^{(6)}(x)$ and $g^{(6)}(x)$?

$f^{(6)}(x) = 0$ since it's 5^{th} degree (+ drops in degree by 1 each time);

$g^{(6)}(x) = -(5)(-6)(-7)(-8)(-9)(-10)x^{-11}$.

3. Let $f(x) = x^{4/3} - 3x^{2/3}$.

$f'(x) = (x^{4/3} - 3x^{2/3})' = (x^{4/3})' - (3x^{2/3})' =$

3.1. Compute the derivative $f'(x)$.

$\frac{4}{3}x^{4/3-1} - 3 \cdot \frac{2}{3}x^{2/3-1} =$

$\boxed{\frac{4}{3}x^{1/3} - 2x^{-1/3}}$

3.2. For which values of x is the derivative $f'(x)$ defined?

$\mathbb{R} - \{0\}$ (or $(-\infty, 0) \cup (0, \infty)$)

3.3. Find an equation for the tangent line to $y = f(x)$ when $x = 1$.

Need $f(1)$ + $f'(1)$; $f(1) = 1 - 3 = -2$; $f'(1) = \frac{4}{3} - 2 = -\frac{2}{3}$

3.4. Using something like Desmos or a graphing calculator, graph $y = f(x)$ for $-6 \le x \le 6$ and this tangent line. Sketch the results below.

$\boxed{y = -2 + (-\frac{2}{3})(x - 1)}$

3.5. Looking at the graph, what behavior do you see where the derivative is undefined?

4. An airplane's height in miles at time t hours is given by the function $h(t) = 5\sqrt{x} - 3\sqrt[3]{x^2}$.

4.1. Write this function as the difference of two power functions.

$h(t) = 5x^{1/2} - 3(x^2)^{1/3} = 5x^{1/2} - 3x^{2/3}$

4.2. What is the function that represents its instantaneous rate of change of height?

$h'(t) = (5x^{1/2} - 3x^{2/3})' = 5 \cdot \frac{1}{2}x^{-1/2} - 3(\frac{2}{3})x^{2/3-1} =$

$\frac{5}{2}x^{-1/2} - 2x^{-1/3}$

4.3. At time $t = 1$ is the plane rising or descending? How fast?

Compute $h'(1) = \frac{1}{2}$ Ascending at $\frac{1}{2}$ mph.

4.4. At time $t = 5$ is the plane rising or descending? How fast?

$h'(5) = \frac{5}{2}\frac{1}{\sqrt{5}} - 2/(5^{1/3})$