Calc 128, Notes #1: Linear things

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I. The most important functions in calculus are the simplest: linear functions of the form *y=mx*, where we call *m* the slope.

a. First of all, what is a function? (We often write this as “*y=f(x)*”, saying “variable *y* is a function of variable *x*”.)

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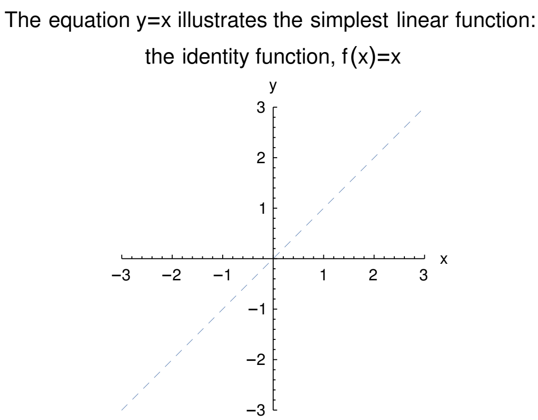

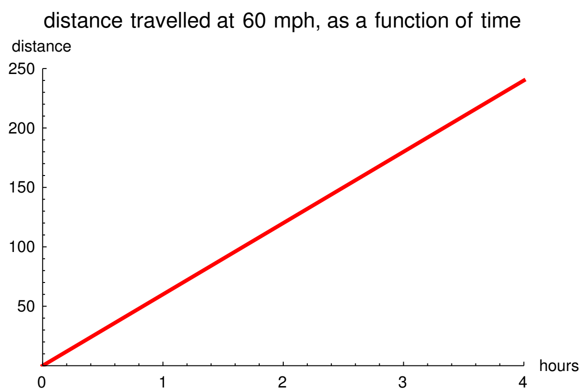

You can recognize a linear relationship -- giving rise to a linear function -- by asking yourself this question (and, if the answer is “yes”, the relationship is linear): “If the size of x doubles, does the size of y double?”

b. One for you to graph: the relationship between distance travelled at a constant speed of 60 mph, and hours travelled.

c. Think of some linear relationships. Test them out on your peers.

c. Think of some relationships that are **not** linear, and test them out on your peers.

II. Affine functions are **almost** linear: y=mx+b is one famous representation of affine functions, called slope-intercept form. Affine functions are **more general** than linear functions, because they allow the intercept b to be non-zero (or we could say that linear functions are **special cases**).

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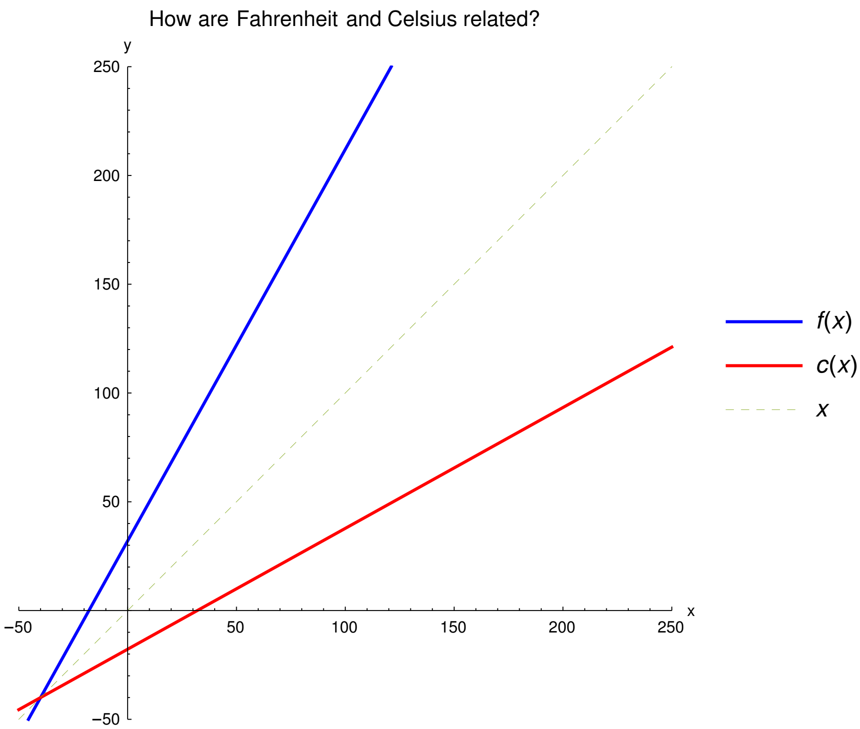

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a. Is it the case that if you double the degrees Fahrenheit, you double the degrees Celsius?
b. State the relationship between Fahrenheit and Celsius: what points do you know of the form "x degrees Fahrenheit equals y degrees Celsius"?
c. Write the equation relating degrees Fahrenheit to degrees Celsius, of the form C=mF+b.
d. Solve for the temperature F as a function of C, and write it also as an equation of slope-intercept form. How are the slopes related?
e. Which temperature has the same value on both scales?
f. Draw the line on the same set of axes above, with the variables switching places (C->F, F->C); what is striking about the two graphs?

III. But something does double, for affine functions: **differences** double. We indicate differences using the Greek letter "Δ"; so "ΔF" means a difference of temperatures, like "$\Delta F = F_1 - F_2$".

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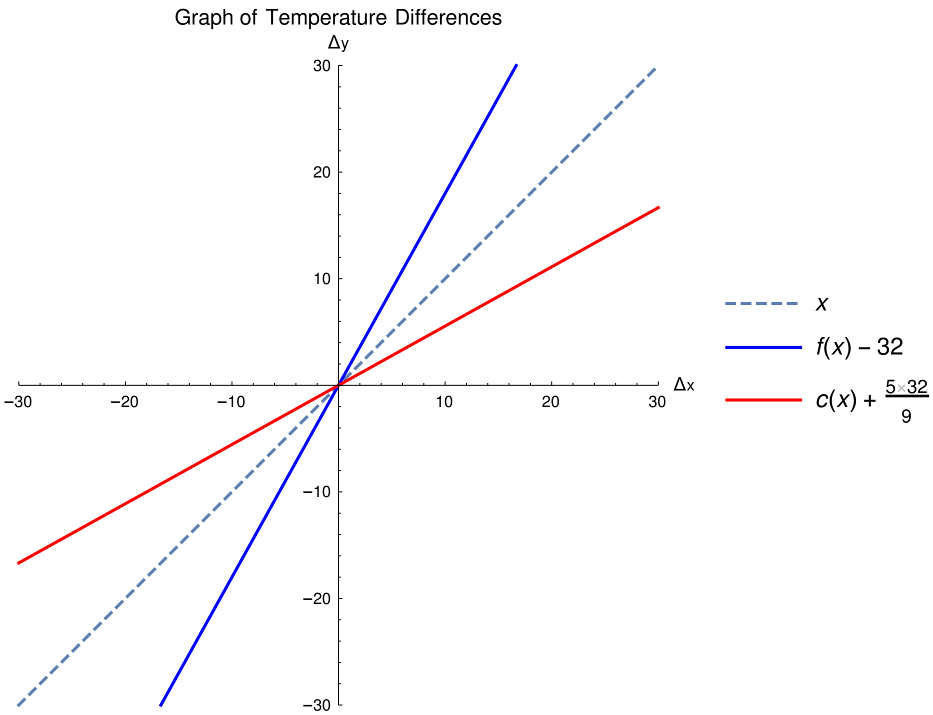


a. If the temperature Celsius rises by 0 degrees, how much does the temperature in Fahrenheit rise?
b. If the temperature Celsius rises by 10 degrees, how much does the temperature in Fahrenheit rise?
c. Draw the linear relationship in **changes** of temperature.
d. Now draw the reverse ("inverse") situation: if the temperature Fahrenheit rises by 10 degrees, how much does the temperature in Celsius rise?

IV. There are three ways we typically write a linear relationship:

a. Slope-intercept form: $F = mC + b$
b. Point-slope form: $F - F_0 = m\,(C - C_0)$
c. Point-point form: $F - F_1 = \frac{F_2 - F_1}{C_2 - C_1}(C - C_1)$

This last form is the one that's pointing us to calculus: notice that

$$F - F_1 = \frac{F_2 - F_1}{C_2 - C_1}(C - C_1) = \frac{\Delta F}{\Delta C}(C - C_1) = m\,(C - C_1):$$

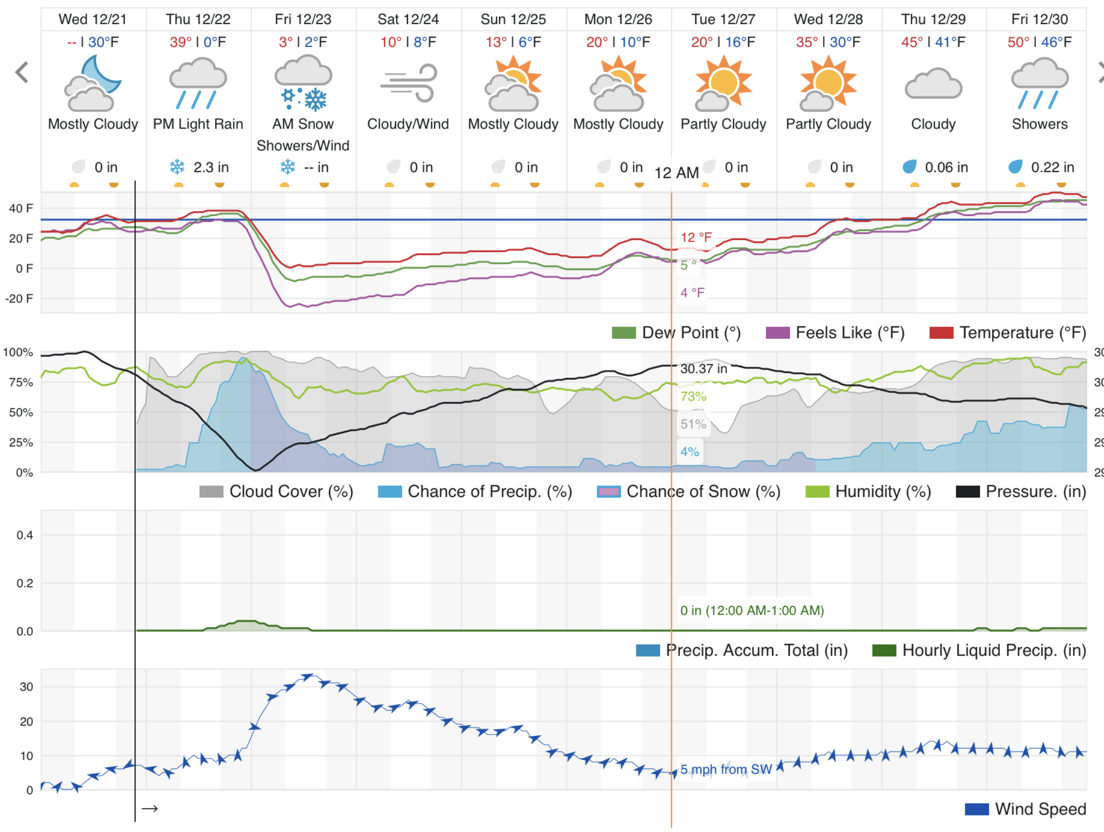
the slope, m, is the same as a "rate of change" (the ratio of the change in F -- ΔF -- to the change in C -- ΔC). In this case, it's the slope of the line joining the points (C_1, F_1) and (C_2, F_2) -- and for linear and affine functions, **any** two distinct points will work, because they're joined by the same line. Write the equation of F as a function of C using each of these three forms:

a. Slope − intercept form : $F = 9/5\,C + 32$
b. Point − slope form : $F - 32 = 9/5\,(C - 0)$
c. Point − point form : $F - 32 = \frac{212 - 32}{100 - 0}(C - 0)$

V. However, more generally (for **non**-linear functions!) the slope varies as we slide along the graph. In any event, however, we simply put our ruler up to the graph, draw the line that fits the curve best at that point, and approximate the slope using the line at that particular point on the graph. Let’s talk about how the weather stacked up during the bomb cyclone:

a. Temperature was not a linear function of time over the ten day period (it’s **non**-linear): what was predicted to happen to **the rate of change** of the temperature?

b. How do you think that these variables are related: temperature, pressure, winds?



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