Homework Section 2.7

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Problems 1-7 at the end of the chapter.

1. Implicit differentiation in a polynomial equation.

Activate

Find dy/dx in terms of x and y if $x^2y - x - 5y - 11 = 0$.

$$\frac{dy}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

In[310]:=
```
soln = Solve[D[x ^2 y[x] - x - 5 y[x] - 11 == 0, x], y'[x]]
```

Out[310]=
$$\left\{\left\{y'[x] \to \frac{1 - 2\,x\,y[x]}{-5 + x^2}\right\}\right\}$$

2. Implicit differentiation in an equation with logarithms.

Activate

Find $\frac{dy}{dx}$ in terms of x and y if $x \ln y + y^3 = 3 \ln x$.

$$\frac{dy}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

In[311]:=
```
soln = Solve[D[x Log[ y[x]] + y[x]^3 == 3 Log[x], x], y'[x]]
```

Out[311]=
$$\left\{\left\{y'[x] \to -\frac{(-3 + x\,\mathrm{Log}[y[x]])\,y[x]}{x\,(x + 3\,y[x]^3)}\right\}\right\}$$

3. Implicit differentiation in an equation with inverse trigonometric functions.

Activate

Find dy/dx in terms of x and y if $\arctan(x^3 y) = xy^3$.

$\frac{dy}{dx} =$ ________________

```
In[312]:= soln = Solve[D[ArcTan[x^3 y[x]] == x y[x], x], y'[x]]
```

Out[312]= $\left\{\left\{y'[x] \to -\frac{y[x]\left(1 - 3x^2 + x^6 y[x]^2\right)}{x\left(1 - x^2 + x^6 y[x]^2\right)}\right\}\right\}$

4. Slope of the tangent line to an implicit curve.

Activate

Find the slope of the tangent to the curve $x^3 + xy + y^2 = 31$ at $(1, 5)$.

The slope is ____.

*(Enter **undef** if the slope is not defined at this point.)*

```
In[313]:= soln = Solve[D[x^3 + x y[x] + y[x]^3 + x y[x] == 31, x], y'[x]]
yp[x_] = y'[x] /. soln[[1]][[1]]
x = 1;
y[x] = 5;
m = yp[x]
Clear[x, y]
localLinearization[x_] := 5 + m (x - 1)
localLinearization[x]
```

Out[313]= $\left\{\left\{y'[x] \to \frac{-3x^2 - 2y[x]}{2x + 3y[x]^2}\right\}\right\}$

Out[314]= $\frac{-3x^2 - 2y[x]}{2x + 3y[x]^2}$

Out[317]= $-\frac{13}{77}$

Out[320]= $5 - \frac{13}{77}(-1 + x)$

5. Equation of the tangent line to an implicit curve.

Activate

Use implicit differentiation to find an equation of the tangent line to the curve $3xy^3 + xy = 16$ at the point $(4, 1)$.

The help (equations) [1] ________________ defines the tangent line to the curve at the point $(4, 1)$.

```
In[321]:= soln = Solve[D[3 x y[x]^3 + x y[x] == 16, x], y'[x]]
yp[x_] = y'[x] /. soln[[1]][[1]]
x = 4;
y[x] = 1;
m = yp[4]
Clear[x, y]
localLinearization[x_] := 1 + m (x - 4)
localLinearization[x]
```

Out[321]= $\left\{\left\{y'[x] \to \frac{-y[x] - 3\,y[x]^3}{x\,(1 + 9\,y[x]^2)}\right\}\right\}$

Out[322]= $\frac{-y[x] - 3\,y[x]^3}{x\,(1 + 9\,y[x]^2)}$

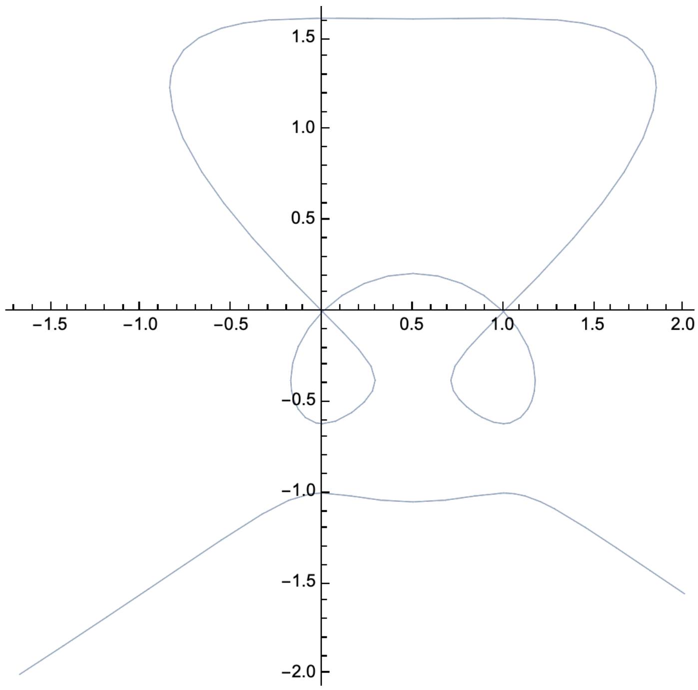
Out[325]= $-\frac{1}{10}$

Out[328]= $1 + \frac{4 - x}{10}$

6. Consider the curve given by the equation $2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$. Find all points at which the tangent line to the curve is horizontal or vertical. Be sure to use a graphing utility to plot this implicit curve and to visually check the results of algebraic reasoning that you use to determine where the tangent lines are horizontal and vertical.

In[329]:=
```
re = ImplicitRegion[2 y^3 + y^2 - y^5 == x^4 - 2 x^3 + x^2, {x, y}];
Region[re, Axes → True]
```

Out[330]=



In[331]:=
```
Solve[D[2 y[x]^3 + y[x]^2 - y[x]^5 - (x^4 - 2 x^3 + x^2) == 0, x], y'[x]]
```

Out[331]= $\left\{\left\{y'[x] \to -\frac{2\left(x - 3 x^2 + 2 x^3\right)}{y[x]\left(-2 - 6 y[x] + 5 y[x]^3\right)}\right\}\right\}$

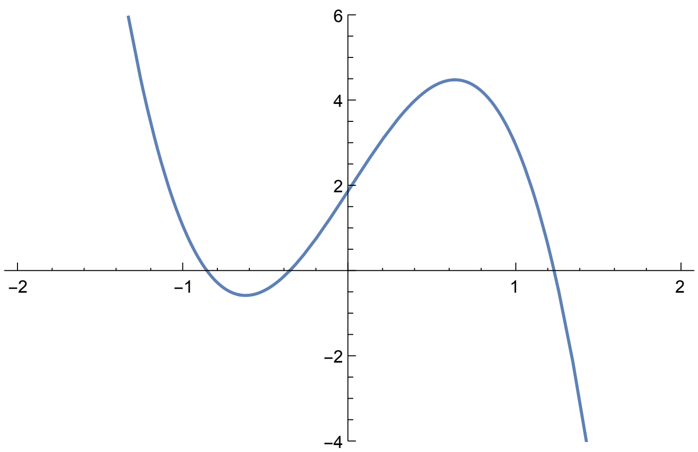
Find the zeros of the denominator:

In[332]:=
```
Solve[-5 y^3 + 6 y + 2 == 0, y]
```

Out[332]= {{y → −0.856…}, {y → −0.379…}, {y → 1.23…}}

In[333]:=
```
Plot[-5 y^3 + 6 y + 2, {y, -2, 2}, PlotRange → {-4, 6}]
```

Out[333]=



7. For the curve given by the equation $\sin(x+y)+\cos(x-y)=1$, find the equation of the tangent line to the curve at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

```
In[334]:= soln = Solve[D[Sin[x + y[x]] + Cos[x - y[x]], x] == 0, y'[x]]
yp[x_] = y'[x] /. soln[[1]][[1]]
x = Pi / 2;
y[Pi / 2] = Pi / 2;
yp[Pi / 2]
Clear[x, y]
localLinearization[x_] := Pi / 2 + (-1) (x - Pi / 2)
localLinearization[x]
```

Out[334]= $\left\{\left\{y'[x] \to \frac{-\text{Cos}[x+y[x]]+\text{Sin}[x-y[x]]}{\text{Cos}[x+y[x]]+\text{Sin}[x-y[x]]}\right\}\right\}$

Out[335]= $\frac{-\text{Cos}[x+y[x]]+\text{Sin}[x-y[x]]}{\text{Cos}[x+y[x]]+\text{Sin}[x-y[x]]}$

Out[338]= -1

Out[341]= $\pi - x$

In[342]:=
```
re = ImplicitRegion[Sin[x + y] + Cos[x - y] == 1, {x, y}];
Show[
 Region[re, Axes → True],
 ListPlot[{{Pi / 2, Pi / 2}}],
 Plot[localLinearization[x], {x, 0, Pi}
 ]
]
```

Out[343]=