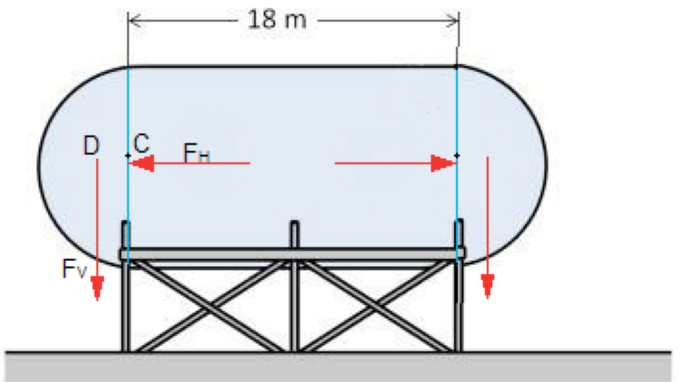
AC3.4.9



Andy Long, Spring 2024

Geometry and costs: using our givens and our equations:

In[399]:=
```
Clear[l, r]
volume[l_, r_] := (4 / 3) Pi r^3 + Pi r^2 l
surfaceAreaTube[l_, r_] := 2 Pi r l (* circumference times length *)
surfaceAreaEnds[r_] := 4 Pi r^2 (* surface area of a sphere *)
cost[l_, r_] := 5 surfaceAreaEnds[r] + 2 surfaceAreaTube[r, l]
cost[l, r]
```

Out[404]= $4\, l\, \pi\, r + 20\, \pi\, r^2$

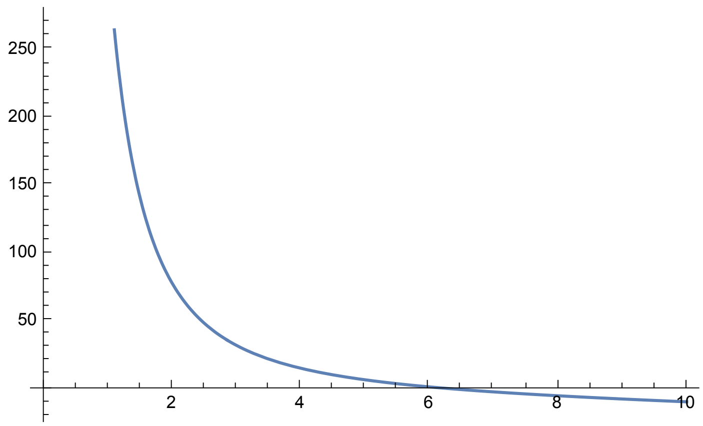
Let's reduce one variable, using the volume constraint:

In[405]:=
```
Clear[l, r]
soln = Solve[volume[l, r] == 1000, l]
l[r_] = l /. soln[[1]][[1]]
Plot[l[r], {r, 0, 10}]
Simplify[cost[l[r], r]]
```

Out[406]= $\left\{\left\{l \to -\frac{4\left(-750+\pi r^3\right)}{3\pi r^2}\right\}\right\}$

Out[407]= $-\frac{4\left(-750+\pi r^3\right)}{3\pi r^2}$

Out[408]=



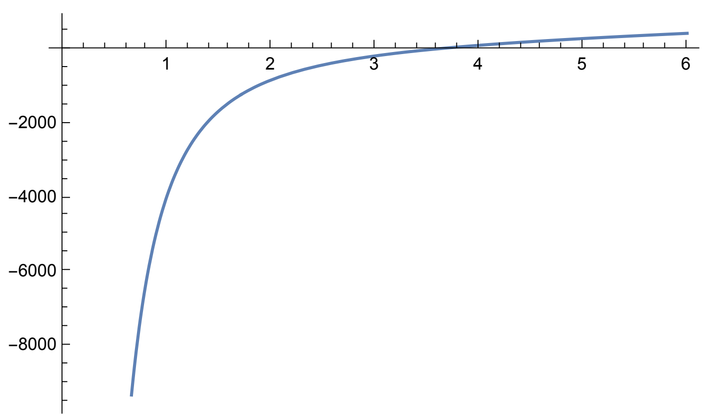
Out[409]= $\frac{4000}{r}+\frac{44\pi r^2}{3}$

Now we differentiate the cost function, and set it equal to 0 to find the critical numbers:

In[410]:=
```
costPrime[r_] = Simplify[D[cost[l[r], r], r]]
Plot[costPrime[r], {r, 0, 6}, AxesOrigin → {0, 0}]
(* As one can see this rational function changes signs only once,
and goes from negative to positive --
 hence it's a minimum at the critical number. *)
solns = Solve[costPrime[r] == 0, r]
N[solns]
(* Only one of them is real: I computed this as *)
root = CubeRoot[1500.0 / (11 Pi)]
(* radius r, in feet *)
l[root]
(* length l, in feet *)
volume[l[root], root]
(* volume at the optimal radius, in feet-cubed -- just a check! *)
cost[l[root], root]  (* in dollars *)
```

Out[410]= $-\frac{4000}{r^2}+\frac{88\pi r}{3}$

Out[411]=



Out[412]= $\left\{\left\{r\to -5\times 2^{2/3}\left(-\frac{3}{11\pi}\right)^{1/3}\right\},\left\{r\to 5\,(-2)^{2/3}\left(\frac{3}{11\pi}\right)^{1/3}\right\},\left\{r\to 5\times 2^{2/3}\left(\frac{3}{11\pi}\right)^{1/3}\right\}\right\}$

Out[413]= {{r → -1.75719343242118 - 3.04354830367983 i},
{r → -1.75719343242118 + 3.04354830367983 i}, {r → 3.51438686484236}}

Out[414]= 3.51438686484236

Out[415]= 21.0863211890541

Out[416]= 1000.

Out[417]= 1707.26793342632

Here's the picture of the cost function, and our optimal solution:

In[418]:=
```
Show[
 Plot[cost[l[r], r], {r, 0, 5}, AxesOrigin → {0, 0}],
 ListPlot[{{root, cost[l[root], root]}}, PlotStyle → {PointSize → Large}]
]
```

Out[418]=