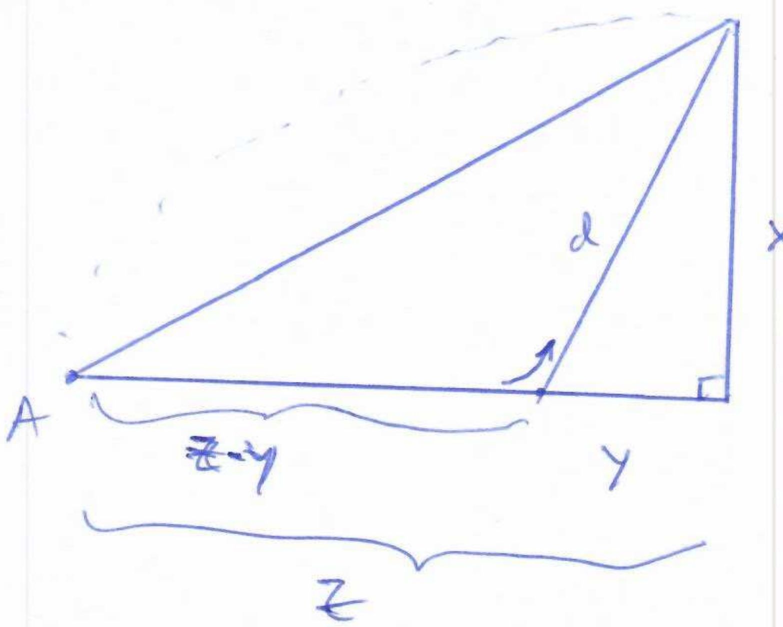

r - running speed

s - swimming speed

Two formulas

$$D = rt$$

$$x^2 + y^2 = d^2$$

Minimize total time:

$$T(y) = T_s(y) + T_r(y)$$

$$= \frac{d}{s} + \frac{z-y}{r}$$

$$T(y) = \frac{\sqrt{x^2+y^2}}{s} + \frac{z-y}{r}$$

Take a derivative of $T(y)$ & set it equal to 0:

$$T'(y) = \frac{1}{s}\left[(x^2+y^2)^{1/2}\right]' - \frac{1}{r}$$

$$= \frac{1}{s}\left[\frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y\right] - \frac{1}{r}$$

$$= \frac{1}{s}\left[(x^2+y^2)^{-1/2}\, y \right] - \frac{1}{r}$$

$$= \frac{1}{s}\left[\frac{y}{\sqrt{x^2+y^2}} \right] - \frac{1}{r}$$

$= 0 \leftarrow$ demand this to find extrema

$$\frac{1}{r} = \frac{1}{s}\,\frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{s}{r}\sqrt{x^2+y^2} = y$$

$$\left(\frac{s}{r}\right)^2 (x^2+y^2) = y^2$$

$$\left(\frac{s}{r}\right)^2 x^2 = y^2\left(1-\left(\frac{s}{r}\right)^2\right)$$

$$\left(\frac{s}{r}\right) x = y\sqrt{1-\left(s/r\right)^2}$$

$$y = \frac{\left(\frac{s}{r}\right)x}{\sqrt{1-\left(s/r\right)^2}} = \frac{s\,x}{\sqrt{r^2-s^2}}$$

$$\boxed{y = \frac{s}{\sqrt{r^2-s^2}}\, x}$$ Theoretical model

We have Elvis data!

$$r \simeq 6.35 \ m/s$$

$$s \simeq 0.90 \ m/s$$

Guess: $m_E \simeq 0.143$

More data:

i	x_i	y_i
1	10.5	2.0
2	7.2	1.0
3	10.3	1.8
⋮	⋮	⋮
35	15.3	3.3

Ask the data!

What's the best fitting line to you?

$y = mx$ experimental m

$$\underbrace{O(m)}_{\text{objective function}} = \sum_{i=1}^{35} \left(y_i - (m x_i)\right)^2$$

(Sum of squared errors over all the data)

Minimize this

$$O'(m) = \sum_{i=1}^{35} 2(y_i - (mx_i)) \cdot (-x_i)$$

$$= -2 \sum_{i=1}^{35} \left(x_i y_i - m x_i^2 \right)$$

$$= 0$$ ← demand this to find an extremum

Split this into two sums,

$$\sum_{i=1}^{35} x_i^2 m = \sum_{i=1}^{35} x_i y_i$$

$$m \sum_{i=1}^{35} x_i^2 = \sum_{i=1}^{35} x_i y_i$$

$$\therefore \boxed{m = \frac{\sum_{i=1}^{35} x_i y_i}{\sum_{i=1}^{35} x_i^2}}$$

$$\boxed{m = 0.17}$$