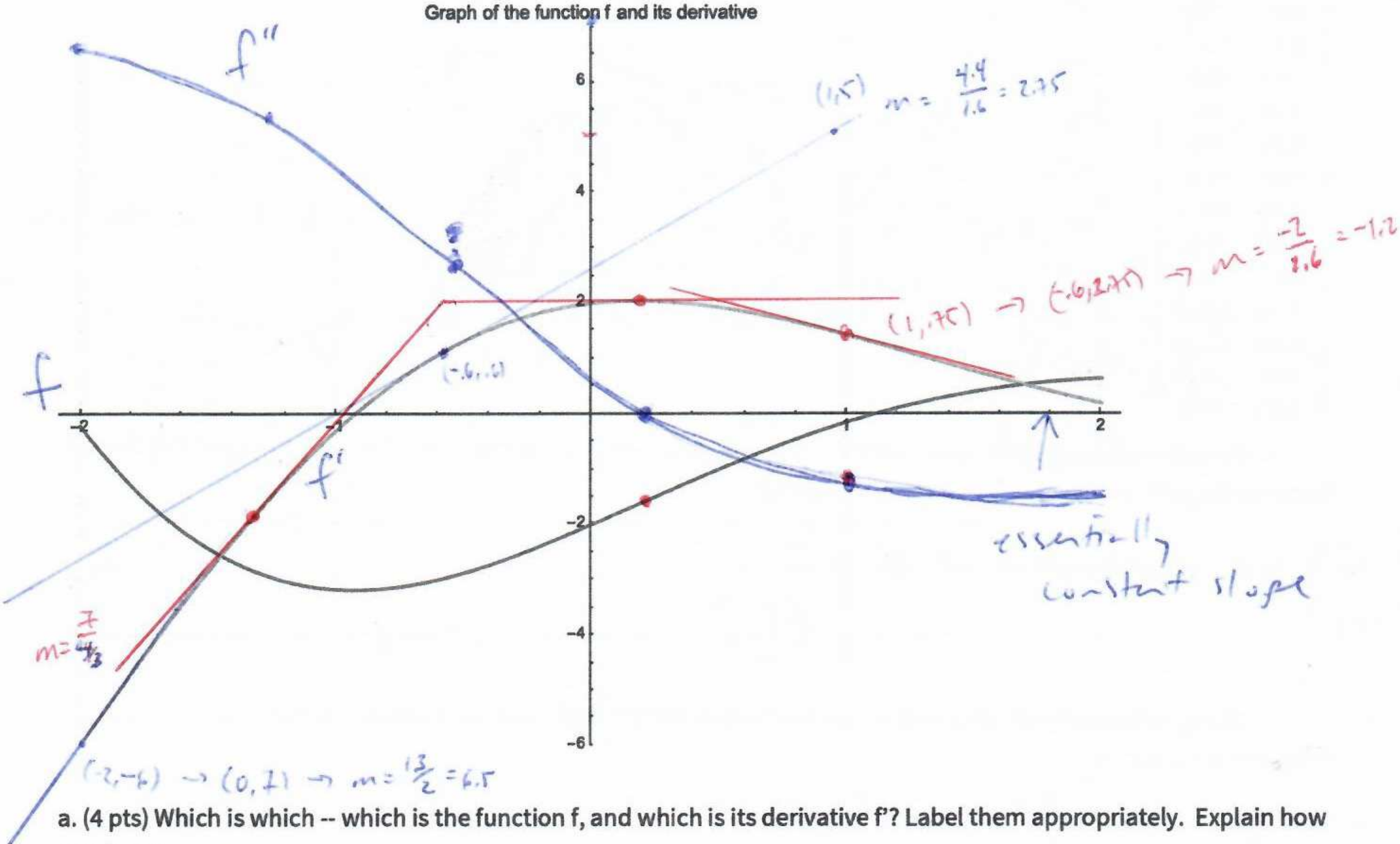
Below is a plot of f and f':



a. (4 pts) Which is which -- which is the function f, and which is its derivative f'? Label them appropriately. Explain how you know.

f' has a zero where f has a minimum

b. (12 pts) Carefully add the second derivative function f''(x) to the graph, using estimates from slopes of tangent lines at several (at least 4) points.

c. (5 pts) Explain how the second derivative you've drawn relates to features of f.

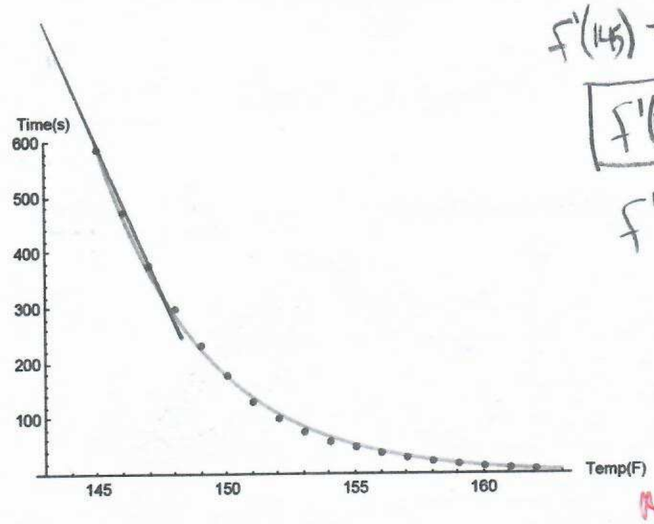
Because f is concave up on (-2, 0.2) (or so..), the second derivative will be + there; & f'' < 0 on (0.2, 2) because f is concave up. There is an inflection point at x = 0.2 (or so).

Problem 2 (30 pts) :

The following data shows the number of seconds (y) one must cook chicken at a chicken's internal temperature (x), in order to safely destroy Salmonella bacterial (call that the "neutralization time"). The plot includes a decaying exponential model, f(Temp)=Time, which I created.

a]

Temp	Time
145	588.
146	474.
147	378.
148	300
149	234.
150	180
151	132.
152	102.
153	78.
154	60
155	49.5
156	39.2
157	31
158	24.5
159	19.4
160	15.3
161	12.1
162	9.6



$$f'(145) = \frac{474-588}{146-145}$$

$$\boxed{f'(145) = -114 \text{ s}/°}$$

$$f'(146) = \frac{378-588}{147-145}$$

$$\boxed{f'(146) = -105 \text{ s}/°}$$

a. (6 pts) Use appropriate difference formulas to compute derivative values for temperatures 145 and 146 degrees **from the data**. Do your work next to the graph above.

b. (2 pts) What are the units of the derivative values?

Seconds per degree temperature (F)

c. (10 pts) Write the local linearization function centered at 146 degrees **in point-slope form**, based on your derivative estimate in part a.

$$L(x) = f(a) + f'(a)(x-a)$$

$$\boxed{L(x) = 474 - 105(x-146)}$$

d. (6 pts) Use the local linearization for 146 degrees to estimate the neutralization time required at x=146.5 degrees.

$$L(146.5°) = 474 - 105(146.5-146)$$
$$= 474 - 52.5$$
$$\boxed{= 421.5 \text{ seconds}}$$

Nice work

Problem 2 (30 pts) :

The following data shows the number of seconds (y) one must cook chicken at a chicken's internal temperature (x), in order to safely destroy Salmonella bacterial (call that the "neutralization time"). The plot includes a decaying exponential model, f(Temp)=Time, which I created.

X Y

Temp	Time
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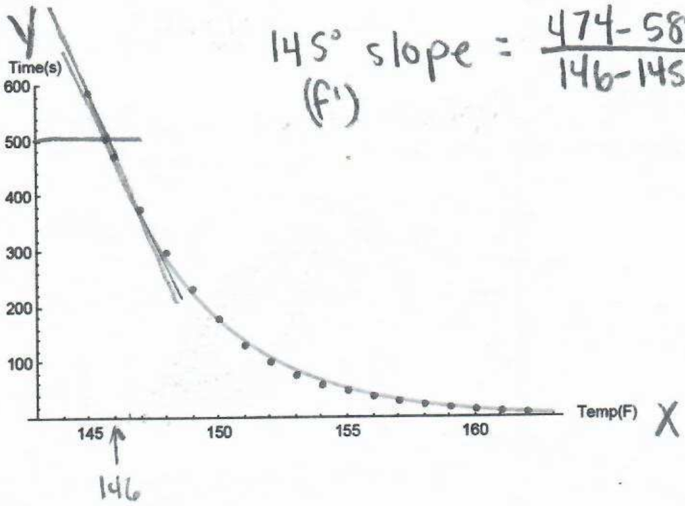

146° slope (f') $= \frac{378-588}{147-145} = \boxed{-105 \text{ s/°F}}$

145° slope (f') $= \frac{474-588}{146-145} = \boxed{-114 \text{ s/°F}}$

a. (6 pts) Use appropriate difference formulas to compute derivative values for temperatures 145 and 146 degrees **from the data**. Do your work next to the graph above.

$f'(146) = -105 \text{ s/°F}, \quad f'(145) = -114 \text{ s/°F}$

b. (2 pts) What are the units of the derivative values?

seconds per °F

c. (10 pts) Write the local linearization function centered at 146 degrees **in point-slope form**, based on your derivative estimate in part a.

$L(x) = (474) + (x-146)(-105)$

d. (6 pts) Use the local linearization for 146 degrees to estimate the neutralization time required at x=146.5 degrees.

$L(146.5) = (474) + (146.5-146)(-105)$

$\boxed{L(146.5) = 421.5 \text{ sec}}$

Excellent

Problem 3 (14 pts) :

The model for the data in the previous problem is a dying exponential function, of the form $f(x) = c\,a^x$ with base a=0.788:

f(Temp) = $(588/0.788^{145})\ 0.788^{\text{Temp}}$ $\left(\frac{588}{0.788^{145}}\right) 0.788^t$

a. (6 pts) Compute its derivative.

$f'g + g'f$

$C=0$

$0.788^x \ln(0.788)$

$= \cancel{0.0788} + 0.788^x \ln(0.788)\left(\frac{588}{0.788^{145}}\right)$

$\boxed{= 0.788^x \ln(0.788)\left(\frac{588}{0.788^{145}}\right)}$

b. (2 pts) Evaluate the derivative at a temperature of 146 degrees.

$f'(146) = 0.788^{146} \ln(0.788)\left(\frac{588}{0.788^{145}}\right)$

$\boxed{= -110.4 \text{ s}/{}^\circ}$

c. (2 pts) How does this derivative approximation compare to your approximation in Problem 1a?

This derivative approximation is less than the approximation in Problem 1a.

d. (4 pts) You made an estimate for the temperature at which the time to neutralization would be exactly 500 seconds (Problem 1f). Evaluate the **model** at that temperature, and compare it to your answer from Problem 1.

$\sqrt[t]{500} = \sqrt[t]{\left(\frac{588}{0.788^{145}}\right)\left(0.788^t\right)}$

$f(145.75) = ?$ -1

$\sqrt[t]{500} = \sqrt[t]{\frac{588}{0.788^{145}}}\ (0.788)$

You're working too hard! :)

Problem 3 (14 pts) :

The model for the data in the previous problem is a dying exponential function, of the form $f(x) = c\,a^x$ with base $a = 0.788$:

$$f(\text{Temp}) = \left(588 \,/\, 0.788^{145}\right)\, 0.788^{\text{Temp}}$$

a. (6 pts) Compute its derivative.

$f' = c a^x \ln(a)$

$f' = (588 / 0.788^{145})\, .788^x \ln(.788)$

b. (2 pts) Evaluate the derivative at a temperature of 146 degrees.

$(588/0.788^{145})\, .788^{146} \ln(.788)$

$\left(588 \div \frac{197^{145}}{250^{145}}\right)\left(\frac{197^{146}}{250^{146}} \times \ln(.788)\right)$

$588 \times \frac{197 \ln(.788)}{250}$

$\rightarrow 294 \frac{197 \ln .788}{125}$

$\frac{57918 \ln(.788)}{125}$

-110.39504

c. (2 pts) How does this derivative approximation compare to your approximation in Problem 1a?

Its fairly close only off by -4 ish. This is more accurate however because the way i estimated took the average rather than the actual

good

d. (4 pts) You made an estimate for the temperature at which the time to neutralization would be exactly 500 seconds (Problem 1f). Evaluate the **model** at that temperature, and compare it to your answer from Problem 1.

-1.5

~~$500 = \left(\frac{588 \cdot 250^{145}}{197^{145}}\right)\left(\frac{\ln(.788)\, 197^x}{250^x}\right)$~~

~~$\left(\quad\right)\left(\frac{-0.238257 \cdot 197^x}{250^x}\right)$~~

$f' = (588 / .788^{145})(.788^{145.75} \ln(.788))$

-117.17042

You're evaluating the derivative

its closer to the estimate done in 2a then it is 3c which makes sense as 2a would be a rough estimate of the point at 145.5

12.5

Problem 4 (25 pts) :

a. (10 pts) Derive the product rule, **starting from the limit definition of the derivative** P'(x)

If f and g are differentiable functions, then their product $P(x) = f(x) \cdot g(x)$ is also a differentiable function, and

$$P'(x) = f(x)g'(x) + g(x)f'(x).$$

Justify each step.

$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$: limit definition

$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)g(x)}{h}$: insert values for P(x)

$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$: insert appropriate form of 0

$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]g(x+h)}{h} + \lim_{h\to 0} \frac{f(x)[g(x+h) - g(x)]}{h}$: separate into 2 limits

$= \lim_{h\to 0} g(x+h) \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + f(x) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$: separate and let h go to 0 for first lim; identify limit definition

$= g(x)\cdot f'(x) + f(x)\cdot g'(x)$

b. (15 pts) Use the rules which we have deduced (e.g. sum, constant multiple, product, power) to differentiate the following functions. Do each step-by-step, justifying the use of each rule:

i. $f(x) = \overset{f}{(x+9)}\,\overset{g}{(x^2 - 3x + 1)}$

$f'(x) = f'g + g'f$ ← product rule

← constant rule ← power rule

$f'(x) = (1+0)(x^2-3x+1) + (2x-3)(x+9)$ $\boxed{f'(x) = 3x^2 + 12x - 26}$

$= (x^2-3x+1) + (2x^2+18x-3x-27)$

sum rule $= (x^2-3x+1) + (2x^2+15x-27)$

ii. $g(x) = 3x^2 - 2x^3 + 7x - 23$

$g'(x) = 6x - 6x^2 + 7$ ← power rule + sum rule

$\boxed{g'(x) = -6x^2 + 6x + 7}$

Great work!

iii. $h(t) = \overset{f}{e^t}\,\overset{g}{(t^2-2t+1)}$

↑ exponent rule special case

← power rule

$h'(t) = e^t(t^2 - \cancel{2t} + 1) + (2t-2)e^t$ ← product rule

$\boxed{h'(t) = e^t(t^2-1)}$

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