MAT128: Exam 3 (Spring 2024): Sections 2.2,2.4-2.7

Name:

Directions: Show our work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answers to each problem (e.g., put a box around them); and clearly separate solutions to each problem from other problems. **Good luck!**

Problem 1: (20 pts) Given the two functions

$$f(x) = x^3 \qquad\qquad g(x) = \ln(x)$$

a. (16 pts) Demonstrate your understanding of the chain rule to compute these derivatives:
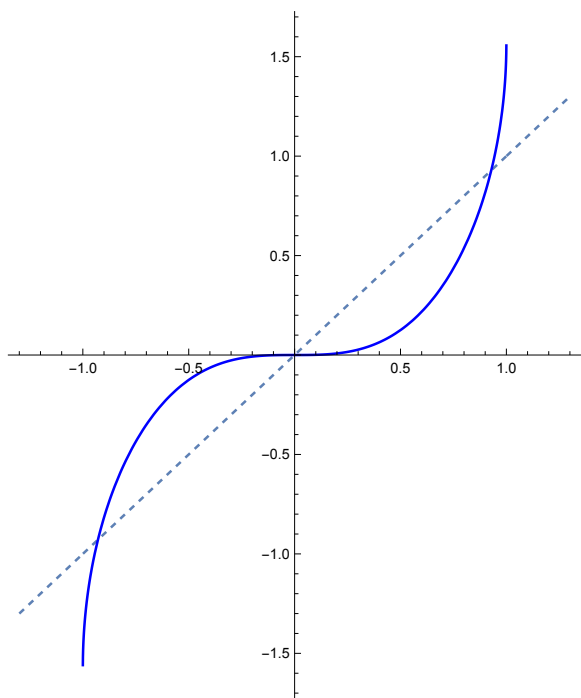
   i. $p(x) = f(g(x))$

   ii. $q(x) = g(f(x))$

b. (4 pts) We used the limit definition of the derivative to discover the chain rule, and here were the first two steps:
$$\lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h} \approx \lim_{h\to 0} \frac{f(g(x) + hg'(x)) - f(g(x))}{h}$$
Explain why that approximation is reasonable.

Problem 2: (20 pts) The graph of an invertible function, $h(x) = \arcsin(x^3)$, is given below:

a. (4pts) **Carefully** draw the inverse function h^{-1} into the graph.

b. (6pts) Find the derivative of $h(x)$ at $(0.5, 0.1253)$

c. (5 pts) Draw the tangent line at $x = 0.5$ onto the graph of h, and find its equation.

d. (5 pts) Draw the tangent line to the graph of the inverse function at $y = 0.5$, and write its equation (you don't need to do any additional calculations!). What do you notice?

Problem 3: (20 pts) In class we considered the folium of Descartes, whose equation and graph are

$$x^3 + y^3 = 6xy \tag{1}$$
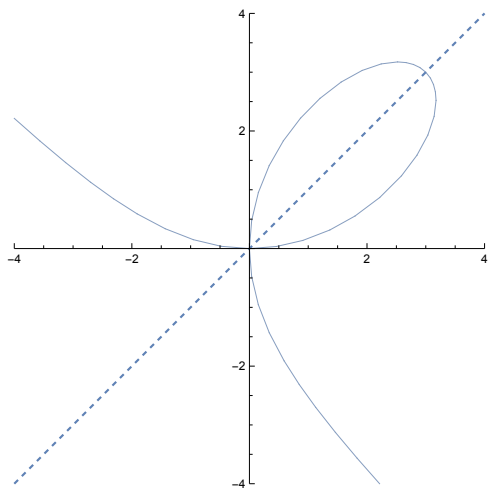
a. (10 pts) Use the space above to compute y' – then write your answer here:

b. Find all points (x, y) where the slope of the tangent line is either 0 or ∞. One place where this happens is at the origin: what's going on there? (1 pt)

There are two other points where we have 0 or ∞ slopes; but if you find **one**, then you've found the **other** (by reflection!). Here's how you do it:

   i. (5 pts) Find the value of y as a function of x that produces zero slope (use your derivative equation!), and then plug that back into the original equation (1).

   ii. (2 pts) Then solve this new equation for x. You've found a pair (x, y).

   iii. (2 pts) Now use symmetry to find the other point.

Problem 4: (10 pts) Find the equation of the tangent line to the graph of

$$f(x) = \frac{3\tan(x)}{1 + \sin(x)}$$

when $x = \dfrac{\pi}{4}$. Write the equation in point-slope form.