Section 1.4 - Derivative as function

This is a cubic - it's derivative will be a quadratic.

1. Consider the function given by $f(x) = x^3$:

a. There are actually several different ways that one typically estimates a derivative at a point using a table of values. The one that we have used so far is called the forward difference (and actually represents an average velocity, of course):

$$AV_{[a,a+h]} = \frac{f(a+h)-f(a)}{h}.$$

Andy's rule: whenever you have two estimates, you have a third: the average of the two!

backward difference: $\frac{f(a)-f(a-h)}{h}$

centered difference: $\frac{f(a+h)-f(a-h)}{2h}$ $= \frac{1}{2}(\text{forward} + \text{backward})$

Fill in the following table, wherever possible (write "NA" if a cell cannot be calculated; the last row of the table will be filled in
later):

circled entries are NA

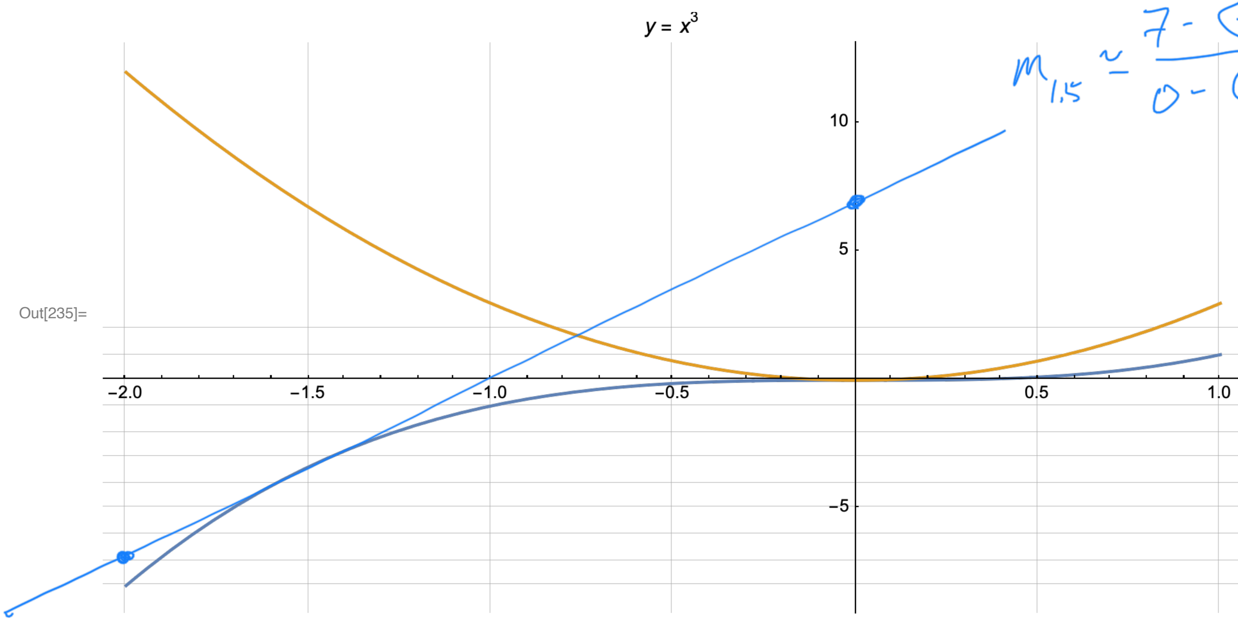
x	-2.	-1.5	-1.	-0.5	0.	0.5	1.
y=f(x)	-8.	-3.375	-1.	-0.125	0.	0.125	1.
forward	9.25	4.75	1.75	0.25	0.25	1.75	4.75
backward	15.25	9.25	4.75	1.75	0.25	0.25	1.75
centered	12.25	7.	3.25	1.	0.25	1.	3.25
f'(x)	12.	6.75	3.	0.75	0.	0.75	3.

b. We can use a graph of the function to estimate the derivative function: just draw tangent lines, and estimate slopes of those tangent lines. Fill in the table below by estimating the slopes from the graph; the last row of the table will be filled in later.

$m_{1.5} \simeq \frac{7-(-7)}{0-(-2)} = \frac{14}{2} = 7$

$\simeq 6.75$

Out[235]=

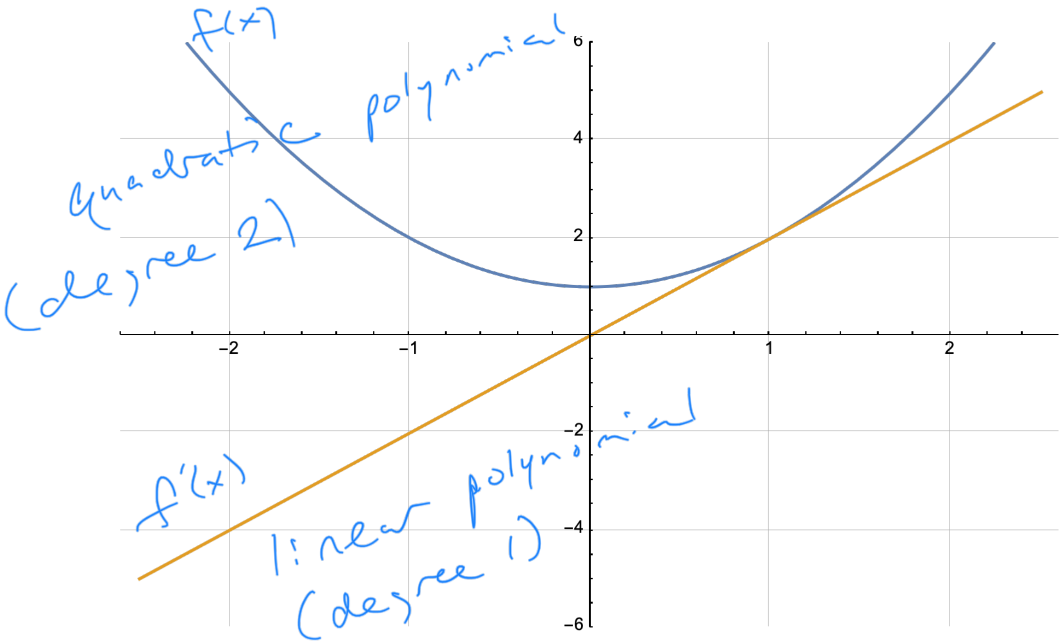


x	-2.	-1.5	-1.	-0.5	0.	0.5	1.
y=f(x)	-8.	-3.375	-1.	-0.125	0.	0.125	1.
slope	12.	6.75 ≃7	3.	0.75	0.	0.75	3.
f'(x)	12.	6.75	3.	0.75	0.	0.75	3.

c. Finally we can use algebra to find the derivative function of f(x). Let’s do so, then fill in the last lines of the tables above with the correct derivative values.

d. Which of the discrete approximations is best? How do your answers compare from your graphical estimates?

2. Given the following graph of y = f(x), draw the approximate graph of y = f'(x).



derivative of a polynomial is a polynomial of degree one lower.

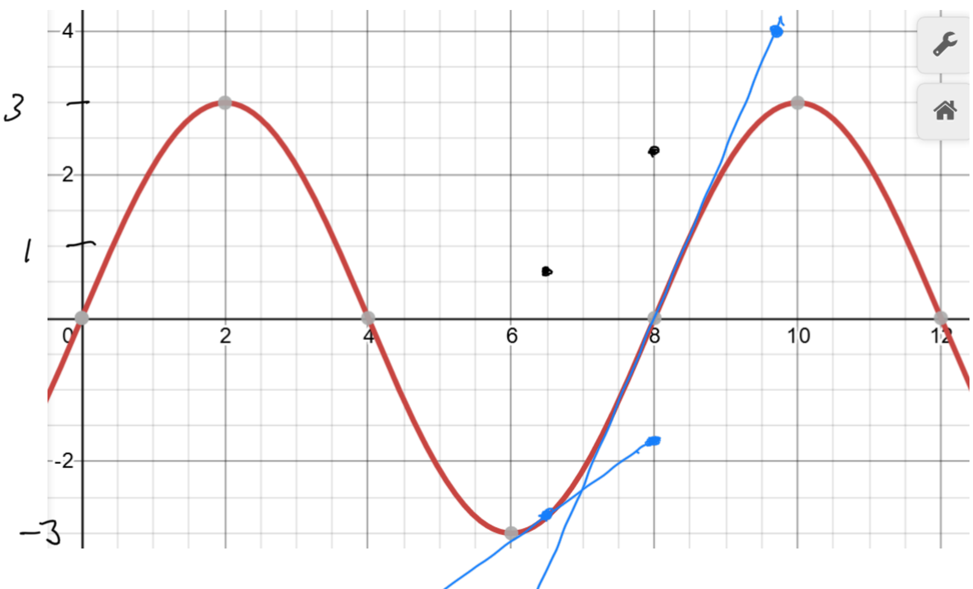
Comment on the symmetry displayed in the function f(x), and how it plays out in the derivative function.

f(x) is even ;

∴ f'(x) is odd .

You can bank on that!

3. Given the following graph of y = f(x), draw the approximate graph of y = f'(x).

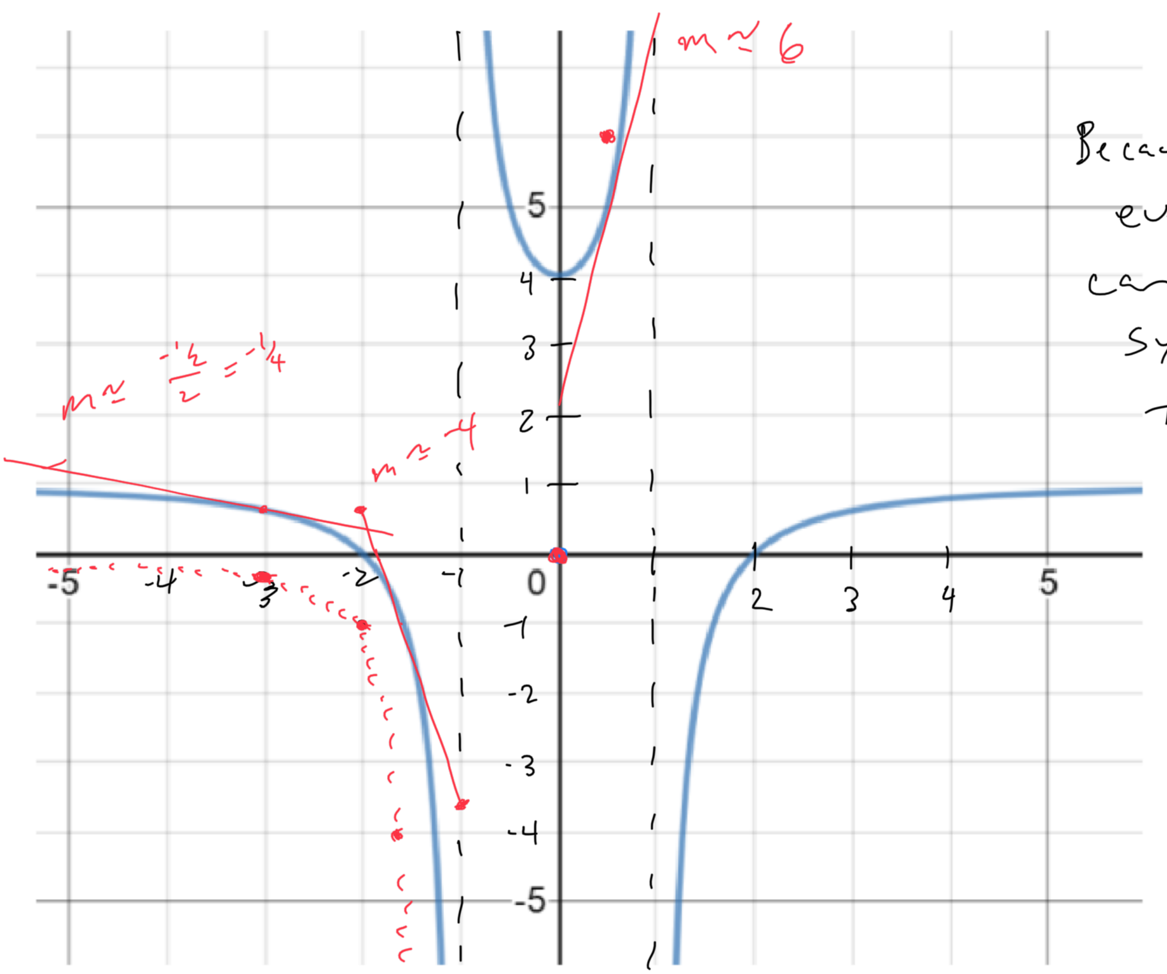


$$m_8 \simeq \frac{4-0}{9.75-8} = \frac{4}{1.75} = \frac{16}{7} \simeq 2.3$$

$$m_{6.5} \simeq \frac{-1.5-(-2.75)}{8-6} = \frac{1.25}{2} = .625$$

4. Given that $y=f(x)=\frac{1}{x+1}$, use the algebraic definition to find a formula for y = f'(x). Show all of your work!

5. Given the following graph of y = f(x) draw the approximate graph of y = f'(x)



Because f is even, we can invoke symmetry to conclude f' will be odd....