Section 1.5 -- More on Derivatives Key

1. a

Out[244]//TableForm=

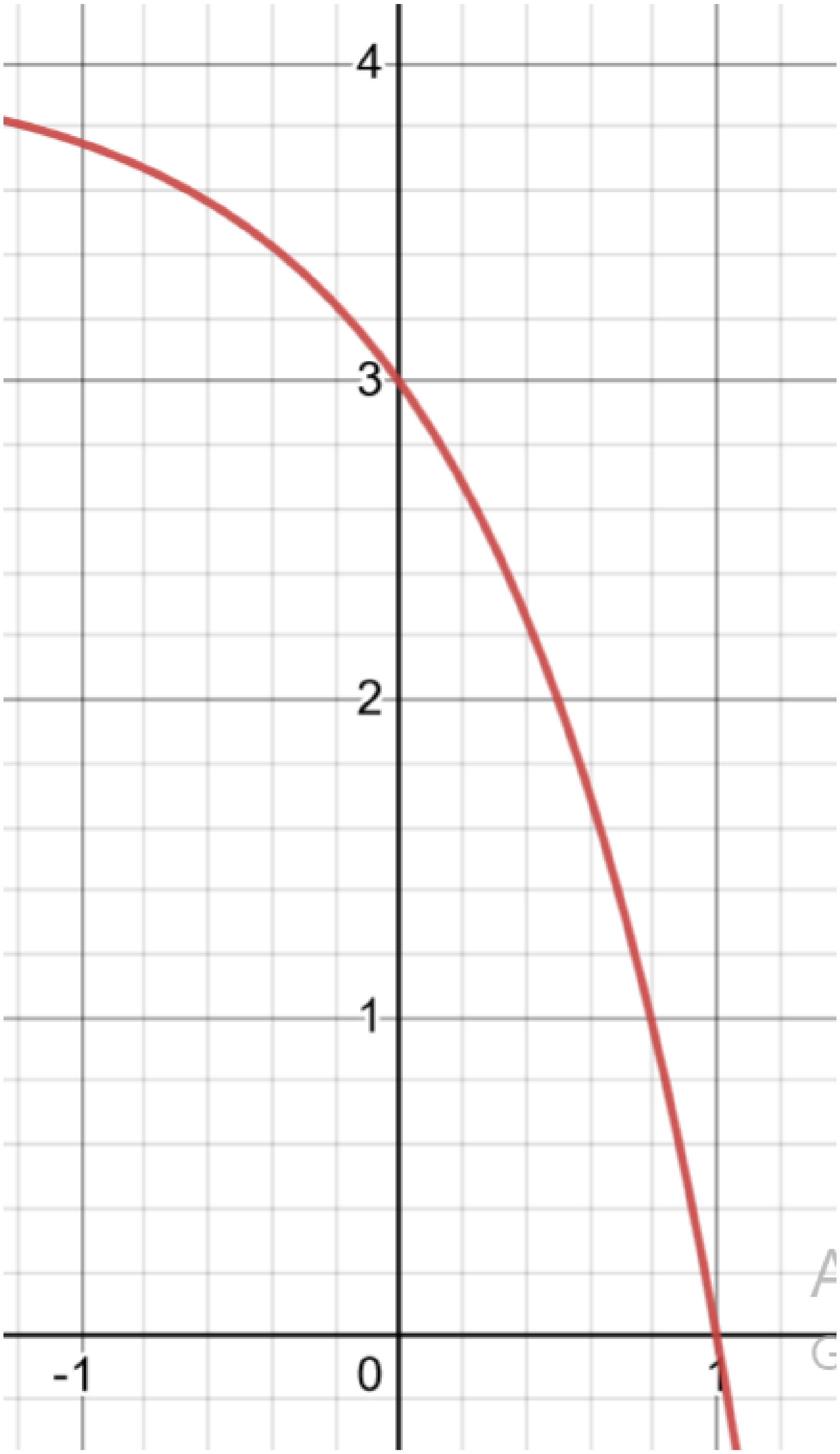
x=days	0	2	4	6
f(x)=height of water (m)	4	6	6	4
f'(x)	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1
type	forward	centered	centered	backward

1. b : units are m/day

1. c : the tank is being filled, is topped off, and then is being depleted.

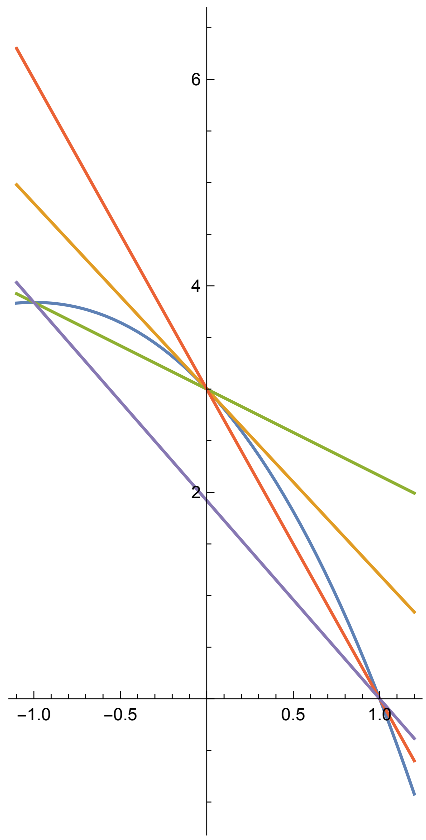
1. d : it looks like the derivative will be about 0 when x = 3.

2. The function f is represented by the following graph.



Draw the tangent line to f at x = 0 and use it to approximate f’(0).

Out[325]=



a. Why would these calculations suggest that the centered difference quotient works better than either the forward or backward differences?

The slopes of the tangent line and the centered secant line look closer than those of the forward or backward secant lines.

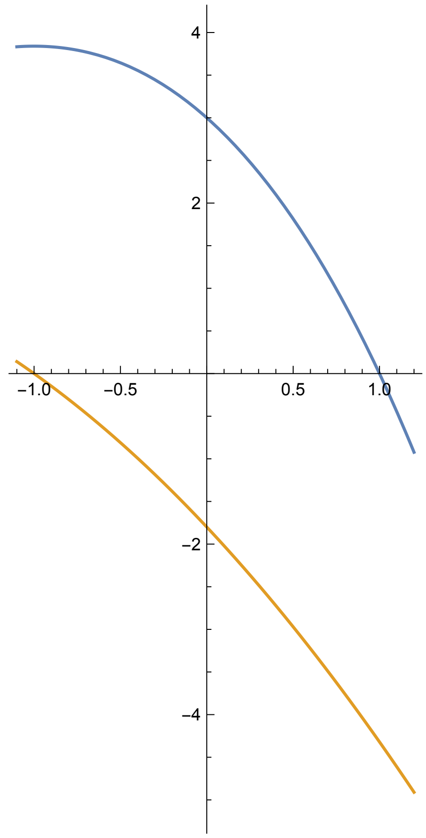
b. Write the equation of the tangent line at x=0, based on your approximation of the slope above.

$$y = 3 - \frac{9x}{5}$$

c. Describe what you can about the derivative function (you might include a crude sketch here).

Since the graph itself looks roughly parabolic, we might expect the graph of the derivative to look roughly linear.

Out[279]=



3. Given that f(x) = 2x^2,

a. Calculate f'(x) using the backward difference quotient, the forward difference quotient and the centered difference quotient.

I'll do the centered difference formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x-h)}{2h} = \lim_{h \to 0} \frac{2(x+h)^2-2(x-h)^2}{2h} = \lim_{h \to 0} \frac{(x+h)^2-(x-h)^2}{h} = \lim_{h \to 0} \frac{x^2+2\,hx+h^2-(x^2+2\,hx+h^2)}{h}$$

$$= \lim_{h \to 0} \frac{2\,hx+2\,hx}{h} = \lim_{h \to 0} 4x = 4x$$

b. Verify that they all give the same answer.

4. A function f is represented by the following table.

a. Fill in the missing cells with the best possible approximation.

Out[320]//TableForm=

x=weeks since Aug 1	0	1	2	3
f(x)=covid hospitalization	600	1000	1200	1300
f'(x)	400	300	150	100
type	forward	centered	centered	backward

b. What are the units of f'(x)?

hospitalizations per week

c. What does f' tell us about the situation?

How quickly beds are filling