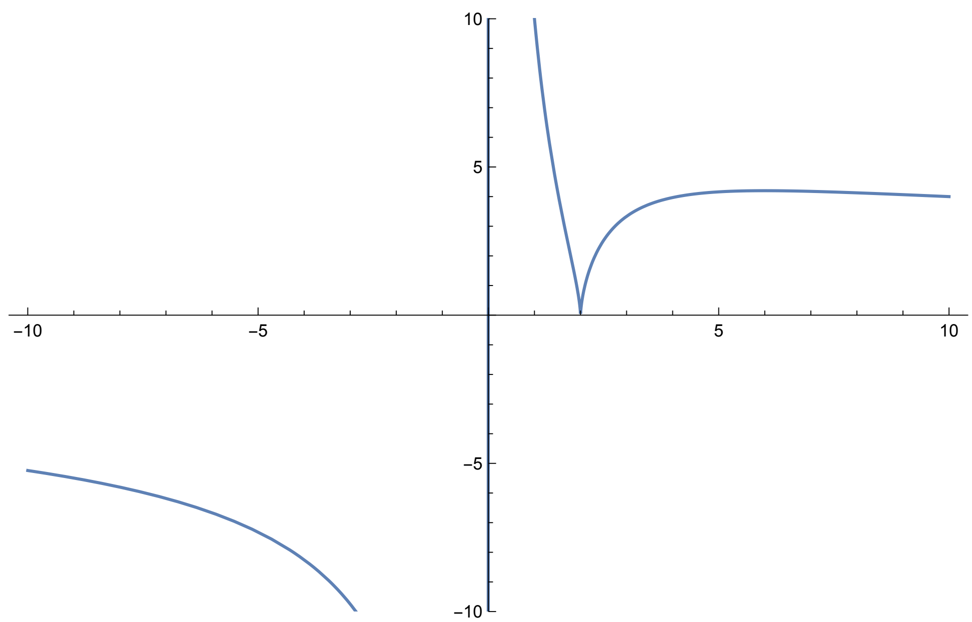
3.1: Critical Points and Extrema

1. Let $h(x) = \frac{10\,(x-2)^{2/3}}{x}$. A computer algebra system gives the derivative as $h'(x) = \frac{10\,(6-x)}{3\,x^2 (x-2)^{1/3}}$.

In[156]:=
```
h[x_] := 10 E^(2/3 Log[Abs[x - 2]]) / x
Simplify[h'[x]]
Plot[h[x], {x, -10, 10}, PlotRange → {-10, 10}]
```

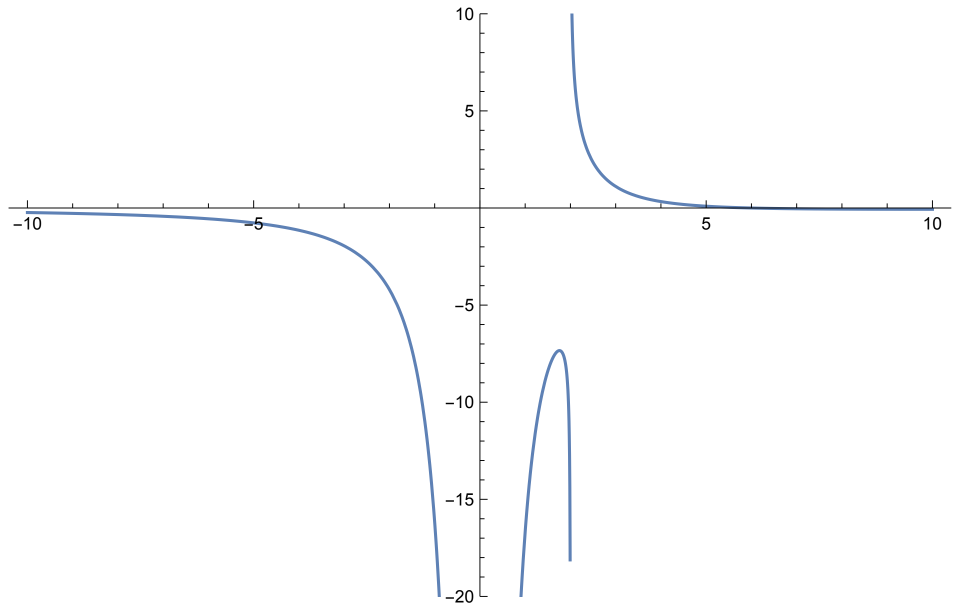
Out[157]= $\frac{10\,(-3\,\text{Abs}[-2+x] + 2\,x\,\text{Abs}'[-2+x])}{3\,x^2\,\text{Abs}[-2+x]^{1/3}}$

Out[158]=



We did this one together in class!

Out[123]=



In[124]:=

1.1. What are the intervals of increase and the intervals of decrease?

1.2. What is the domain for the original function $h(x)$? What is the domain for the derivative $h'(x)$?

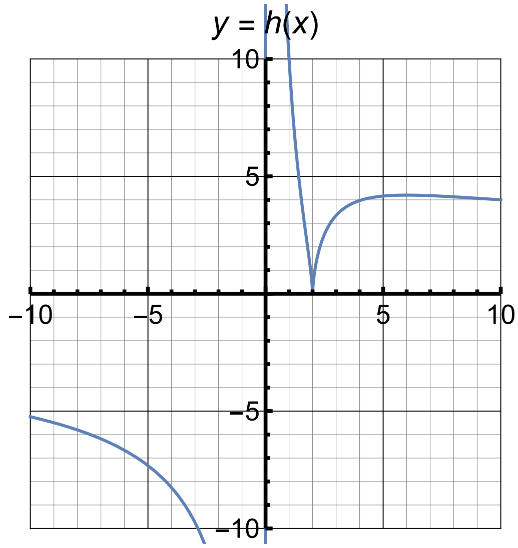
Domain of $h(x)$:

Domain of $h'(x)$:

1.3. Find all the critical numbers of $h(x)$, and for each critical number determine if it is a local maximum, a local minimum, or neither.

1.4. Plot the graph in Desmos or a graphing calculator. Sketch the results below. Does your results in 1.1, 1.2 and 1.3 agree with the graph?

Out[132]=



2. Let $f(x) = \frac{(x-1)^{1/3}}{x+2}$. A computer algebra system gives the derivative as $f'(x) = \frac{5-2x}{3(x+2)^2 (x-1)^{2/3}}$.

2.1. What are the intervals of increase and the intervals of decrease?

Studying the sign of f', the sign is determined by the numerator alone. So 0 at $5/2$, positive to the left (↗) & negative at right ↘

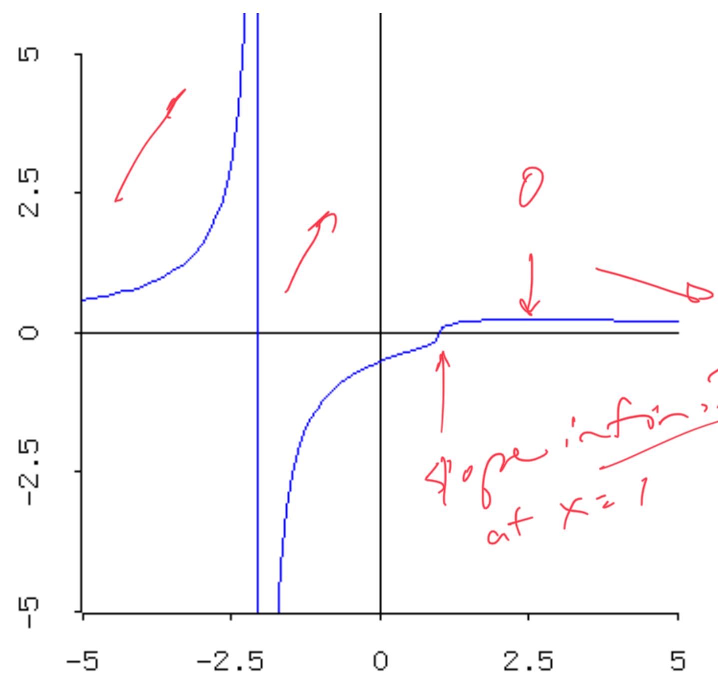
2.2. What is the domain for the original function $f(x)$? What is the domain for the derivative $f'(x)$?

Domain of $f(x)$: $\mathbb{R} - \{-2\}$

Domain of $f'(x)$: $\mathbb{R} - \{-2, 1\}$

2.3. Find all the critical numbers of $f(x)$, and for each critical number determine if it is a local maximum, a local minimum, or neither. (Check your results with a graph from Desmos or a graphing calculator. Do they agree?)

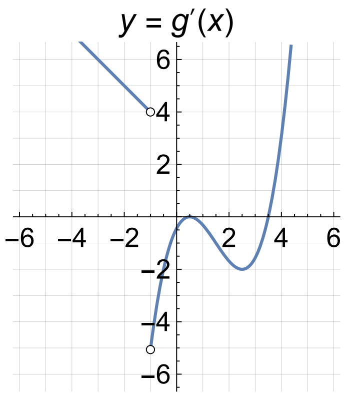
$x = \frac{5}{2}$ is a maximum (local).
The derivative is not defined at $x = -2$, & at $x = 1$ (inflection point)



⇒ domain of f' does not contain 1.

3. Function $g(x)$ has domain $(-\infty, \infty)$. The graph shown below is of the derivative $g'(x)$ (NOT $g(x)$). However, use the graph to answer questions about the original function $g(x)$.

Out[126]=



3.1. What are the intervals of increase for $g(x)$. (What does that mean for $g'(x)$?)

Where $g'(x)$ is positive $(-\infty, 4) \cup (3.5, \infty)$

3.2. What are the intervals of decrease for $g(x)$. (What does that mean for $g'(x)$?)

When $g'(x)$ is negative $(-4, 0.5) \cup (0.5, 3.5)$

3.3. Find all the critical points for $g(x)$, and for each one determine if it is a local maximum, a local minimum, or neither.

$x = -1$ not sure - depends if g exists at $x = -1$.

$x = 0.5$ (inflection point) : neither max nor min

$x = 3.5$ (local min ↘ 0 ↗)