Section Summary: 1.5

a. Definitions

- **limit of $f(x)$ as x approaches a**: Suppose function $f(x)$ is defined when x is near the number a (this means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

  $$\lim_{x \to a} f(x) = L$$

  if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a but not equal to a.

  We say that "the limit of $f(x)$ as x approaches a equals L." The intuitive idea is that in the neighborhood of a, the function f takes on values close to L.

- **left-hand limit of $f(x)$ as x approaches a from the left**:

  $$\lim_{x \to a^-} f(x) = L$$

  if and only if the limit exists from the left – in the neighborhood of a, but only from the left-hand side, where $x < a$.

- **right-hand limit of $f(x)$ as x approaches a from the right**:

  $$\lim_{x \to a^+} f(x) = L$$

  if and only if the limit exists from the right – in the neighborhood of a, but only from the right-hand side, where $x > a$.

- **infinite limits for $f(x)$ as x approaches a**:

  $$\lim_{x \to a} f(x) = \infty$$

  means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a (but not equal to a).

  Similarly we can define

  $$\lim_{x \to a} f(x) = -\infty$$

  and one-sided limits such as

  $$\lim_{x \to a^-} f(x) = \infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \infty$$

  In any of these cases, we define a **vertical asymptote** of the curve $y = f(x)$ at $x = a$.

b. Theorems

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

c. Summary

Limits concern what happens as we approach a point (but don't actually arrive there). We're sniffing about in the close proximity of a point. We never learn what is actually happening at the point itself – that's not our objective.

In this section we see a number of examples of how a function may behave (or even misbehave) in the proximity of a point a. The function may approach the value of $f(a)$ as $x \to a$; it may oscillate wildly as $x \to a$; it may tend to one value on the left, and another on the right (leading to a jump in the function); functions may even tend to infinity or negative infinity. All kinds of exciting things may transpire. This section catalogs them, and gives us a hint as to how to investigate them.