Section Summary: 3.5

a. Definitions

slant asymptote: if

$$\lim_{x\to\infty}[f(x) - (mx+b)] = 0$$

or

$$\lim_{x\to-\infty}[f(x) - (mx+b)] = 0$$

then the line $y = mx + b$ is called a slant asymptote. Intuitively, the function f approaches the line as $x \to \infty$.

In particular, there will be a slant asymptote for any rational function whose numerator polynomial degree exceeds the denominator polynomial degree by 1 (e.g. quadratic over linear, or cubic over quadratic, etc.).

b. Properties/Tricks/Hints/Etc.

Guidelines for sketching a curve:

i. Domain - find it!

ii. Symmetry - even and odd functions; functions that have symmetry about some displaced point; periodicity

iii. Intercepts - crossings of the axes. The y-intercept is the point $(0, f(0))$ (if 0 is in the domain); the x-crossings are the roots of the equation: solve $f(x) = 0$ for these special values of x.

iv. Asymptotes - vertical, horizontal, slant

v. Intervals of increase or decrease - use the Increasing/Decreasing Test, based on the sign of the first derivative.

vi. Local maxima and minima - use the first or second derivative tests.

vii. Concavity and points of inflection - use the second derivative and the concavity test.

viii. Compute some points on the curve, especially any that are easy to calculate.

ix. Sketch the curve - sketch asymptotes as dashed lines; plot any known points on the curve (e.g. intercepts); finish by connecting the points in accord with the information above.

c. Summary

We are fortunate to have the calculators and computers we have to help us plot functions. As noted in the book, however, and as we've seen in class, calculators are dumb devices which merely plot many points and then connect them with line segments. This leads to a risk of confusing us rather than enlightening us.

We've just encountered some analytical tools which enable us to appreciate much of the qualitative behavior of a function, such as the first and second derivative tests, the concavity tests, the increasing/decreasing test, etc., and we can use these to gain the valuable intuition about a function (even if it is much harder for us to plot 1000 points!). The good news is that humans are still good for something! Computers haven't replaced us, yet.

In addition, we see in this section that there are other types of asymptotic behavior: in particular, the notion of a slant asymptote is introduced (which is a non-horizontal line which the graph of a function approaches as $x \to \infty$, or $x \to -\infty$). This can be generalized to other kinds of curves: e.g. a rational function whose numerator is of degree 3 divided by a denominator of degree 1 will look like a quadratic (degree 3-1=2) far from the origin, as $x \to \infty$.