Section Summary: 4.4

a. Definitions

- **indefinite integral**: represents the family of antiderivatives of a function, so that

  $$\int f(x)\,dx = F(x) + C \Longrightarrow F'(x) = f(x)$$

  Notice that the author doesn't include the C. Yet he goes on to say that "we can regard an indefinite integral as representing an entire *family* of functions (one antiderivative for each value of the constant C)." So I like to put the C in there, and think of $F(x)$ as a *particular* antiderivative.

  Remember that a definite integral has limits of integration, so that it represents a "definite area" (which is a number); the indefinite integral represents a function. One way we represent this is as follows:

  $$\int_a^b f(x)dx = \int f(x)dx\Big]_a^b$$

  We might read this as "the integral of f from a to b is equal to an antiderivative evaluated at b minus the same antiderivative evaluated at a."

b. Theorems

Total Change Theorem: The integral of a rate of change is the total change:

$$\int_a^b F'(x)dx = F(b) - F(a)$$

We add up all the small changes in the function F from a to b to discover what the total change was in the function F.

c. Properties/Tricks/Hints/Etc.

- "We adopt the convention that when a formula for a general indefinite integral is given, it is valid only on an interval." Thus, we write

  $$\int \frac{1}{x^2}dx = -\frac{1}{x} + C$$

  even though the general antiderivative of $\frac{1}{x^2}$ is best given by

  $$F(x) = \begin{cases} -\frac{1}{x} + C_1 & \text{if } x < 0 \\ -\frac{1}{x} + C_2 & \text{if } x > 0 \end{cases}$$

d. Summary

The definition of the indefinite integral emphasizes the close relationship between differentiation and integration. These two processes are inverses of each other, in much the same way that the square root is the inverse process of the square.

The indefinite integral is a function (or, more accurately, a family of functions), whereas a definite integral is a number (representing a fixed area). Furthermore, notice that the "dummy variable of integration" (x) is used as the variable of the antiderivative, e.g.

$$\int f(x)\, dx = F(x) + C$$

This is a point of confusion for many students, especially after I've spent all this time telling you that the variable under the integral sign is "integrated out", and "disappears".

As mathematicians we are always tempted to reuse our favorite symbols, and that's partly what's happening here. But we also reuse the symbol x because the indefinite integral is playing a slightly different role than the definite integral: the indefinite integral is just a short-hand for the antiderivatives of $f(x)$, which will also be functions of x ($F(x) + C$) – so, in this case, we kind of like to use the same variable on both sides of the equation.

Remember, however, that "a rose by any other name would smell as sweet", and that $F(t)$, $F(x)$, $F(nose)$, and $F(rose)$ all represent the same thing. These variables are just place holders:

f() would smell as sweet....