Section Summary: 3.9 - Anti-derivatives

a. Definitions

- **antiderivative** - A function F is called an antiderivative of f on an open interval I if $F'(x) = f(x)$ for all x in I.

  That is, f is the derivative of F, so F is an antiderivative of f: more than one function has f for its derivative. The relationship is not "monogomous"!

- **differential equation** - an equation involving a function f and its derivatives.

  The point of such equations is that we are seeking a function whose derivatives satisfy a particular relationship with it; for example, a function whose second derivative is equal to the negative of the function itself (whose acceleration is equal to its position, but opposite in sign). The sine function satisfies this constraint.

- **direction field** - a "graph" indicating the trend in a function, based on its derivative.

b. Theorems

If F is an antiderivative of f on an open interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

I like to say that the relationship is not "monogomous": f has lots of antiderivatives, whereas each function has its unique derivative. How sad, that a function f would cheat on its antiderivatives, whereas the antiderivatives are pledged to f (shared by so many others). What a cad!

c. Properties/Tricks/Hints/Etc.

The reference section at the end of the text contains a short table of antiderivatives. In the old days, whole huge books of such antiderivatives were published and frequently consulted by mathematicians – still are in some cases, although computer software has essentially made them obsolete.

There are certain standard anti-derivatives that one needs to be aware of: for example, the anti-derivative family of x^n is

$$\frac{x^{n+1}}{n+1} + C$$

d. Summary

An antiderivative of f is simply a function which has f for its derivative. We've seen that lots of functions have f for a derivative, but that they vary by very little: a constant.

This chapter has us "thinking backwards", and this is a skill which takes times and practice to master. Whereas differentiation is essentially a mechanical exercise, antidifferentiation may require insight, and broad exposure to lots of different functions. One asks oneself, "do I know a function whose derivative looks like this?" Starting from the derivative(s), and perhaps initial conditions (e.g. the value of the function at a point), we may be able to use direction fields to trace a pretty good representation of the function, and then deduce a formula from that.