Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Note**: you must skip one of the problems marked "skippable". Write skip across it, so I'll easily know which one. **Good luck!**

Problem 1 (skippable, 10 pts): Demonstrate your differentiation prowess (simplify where possible):

a. Quotient rule: $f(x) = \dfrac{\cos(x)}{x^2 - 3}$

$$f'(x) = \frac{(x^2-3)(-\sin x) - (\cos(x))(2x)}{(x^2-3)^2}$$

✓

b. Chain rule: $g(x) = \cos(x^2 - 3)$

$$f'(x) = (-\sin(x^2-3))(2x)$$
$$= -2x\sin(x^2-3)$$

✓

c. Chain rule: $p(x) = (\cos(x))^2 - \cancel{3}$

$-\sin x$ $\quad 0$

$$f'(x) = -2\cos x \sin x$$

✓

d. Product rule: $q(x) = \cos(x)(x^2 - 3)$

$$f'(x) = (-\sin x)(x^2-3) + (\cos x)(2x)$$
$$= 3\sin x - x^2\sin x + 2x\cos x$$

✓

10

Problem 2(10 pts): Assume that f and g are differentiable at x, and that $p(x) = f(x) + g(x)$.

a. (7 pts) Use the limit definition of the derivative to demonstrate the sum rule – that is, that

$$p'(x) = f'(x) + g'(x)$$

$$p'(x) = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$ -5 parentheses are essential

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$f'(x)$ $g'(x)$

$p'(x) = f'(x) + g'(x)$ // ✓

Well done, otherwise

Problem 2(10 pts): Assume that f and g are differentiable at x, and that $p(x) = f(x) + g(x)$.

a. (7 pts) Use the limit definition of the derivative to demonstrate the sum rule – that is, that

oh boy i forgot the limit definition

Uh oh...

$$p'(x) = f'(x) + g'(x)$$

$\lim_{x \to 0}$ (x+h)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\cancel{\lim_{g \to 0} \frac{f(x+g) - f(g)}{g}}$$

You remembered!

A limit is an approximation of a derivative, so...

Now to use it ....

$$\cancel{\frac{f'(x+g) - f'(g)}{g}} = f'(x) + g'(x) = 0$$

$$\cancel{p'(x) - f'(x) = g'(x)}$$

$$\cancel{\frac{f'(x) + g'(x) - f'(g)}{g}} = 0 \qquad \cancel{g'(x) = \frac{f(x+g) - f(g)}{g}}$$

$$\cancel{f'(x) + g'(x) - f'(g) = 0} \qquad \cancel{p'(x) - f'(x) = \frac{f(x+g) - f(g)}{g}}$$

-3

b. (3 pts) Suppose that f and g above are both differentiable on $[a, b]$, where a and b are real numbers. What does the Extreme Value Theorem guarantee about the function p?

function P, as a combination of two differentiable functions, is differentiable as itself. A differentiable function is continuous; the EVT guarantees that for any continuous function with a defined ~~domain~~ closed interval [a, b] there exists both a global maximum and minimum.

Good!

7

Problem 3(10 pts): Related Rates: A ferris wheel spinning at 90 mph flies off its stand and hits an ice cream truck traveling 30^o north-by-northwest at 12 ft/s, as an extension ladder slides off the truck into the southbound lane at half that speed. The driver of the truck (a mathematician) dies while considering the following related rates problem:

An oil slick is spreading out in a circle from an oil drilling platform, which is leaking oil into a calm sea. The slick is one cm thick. The oil is spilling at the rate of 1000 liters[1] per second.

a. (2 pts) What is the volume of the slick (a cylinder of circular cross section of radius r and height h)? (Draw it. I'll sell the answer to you, if you can't figure it out.)

const film thick (h)

1000 L/s

$V = \pi r^2 h$

$= V_{cm^3} = \pi r^2 \cdot 1$ ✓

$A_{cm^2} = \pi r^2$

b. (4 pts) How fast is the slick's radius increasing when the slick is 25 meters in radius?

constant

$(V = \pi r_{cm}^2 \cdot h)$

know this ✓

$\frac{dV}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

$1000000\ cm^3/s = 2\pi \frac{dr}{dt} r$

$\frac{1,000,000\ cm^3/s}{2\pi} = 2500\ cm \cdot \frac{dr}{dt}$ ✓

$\frac{10,000\ cm^3/s}{50\pi\ cm^2} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{200}{\pi} \frac{cm}{s}$

good unit work!

c. (4 pts) How fast is the slick's area changing at the very same moment?

$A = \pi r^2$

200 cm/s

$\frac{dA}{dt} = 2\pi \frac{dr}{dt} \cdot (r)$ 2500 cm

rats!

$\frac{dA}{dt} = 1,000,000\,(\pi)\ cm^2/s$ ~ ✓

---

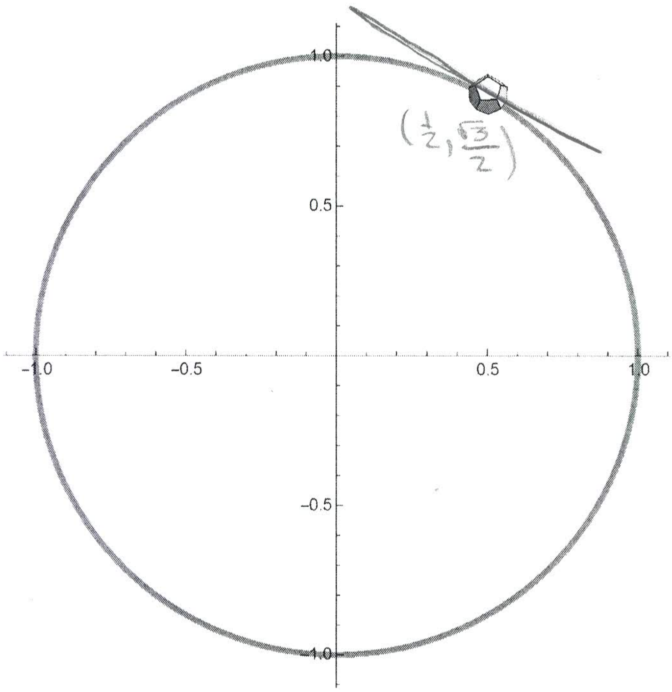
[1] A liter is 1000 cm^3.

10

Problem 4 (skippable, 10 pts): Consider the unit circle ($x^2+y^2=1$) at the point $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

Impl. Diff. ∴ x & y are related, but how

a. (6 pts) Demonstrate implicit differentiation to find the equation of the tangent line at that point. Sketch in your tangent line.



$x^2+y^2=1$

$(x^2+y^2)'=(1)'$ ★ differ. each side

$2x+2y\cdot y'=0$

$2y\cdot y'=-2x$

$y'=\frac{-2x}{2y}=\frac{-x}{y}$

$y'=\frac{-1/2}{\sqrt{3}/2}=\frac{-1}{2}\cdot\frac{2}{\sqrt{3}}=\frac{-1}{\sqrt{3}}=m$ ✓

$(y-y_1)=m(x-x_1)$ ★ slope pt form

$y-\frac{\sqrt{3}}{2}=\frac{-1}{\sqrt{3}}\left(x-\frac{1}{2}\right)$

$y=\frac{-1}{\sqrt{3}}\left(x-\frac{1}{2}\right)+\frac{\sqrt{3}}{2}$

(4 pts) Now do the same problem **explicitly**:

a. Solve the equation of the unit circle for y as the appropriate **explicit** function of x.

$x^2+y^2=1$

$\sqrt{y^2}=\sqrt{1-x^2}$

$y=\sqrt{1-x^2}$ ✓

b. Differentiate your expression $y(x)$ to obtain the slope of the tangent line at $x=\frac{1}{2}$.

$y'=\left(\sqrt{1-x^2}\right)'=\left((1-x^2)^{1/2}\right)'=\frac{1}{2}(1-x^2)^{\frac{-1}{2}}(-2x)=\frac{1}{2}(-2x+2x^3)=-x+x^3$

-1

$y'\left(\frac{1}{2}\right)=-\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^3=\frac{-1}{2}+\frac{1}{8}=\frac{1}{8}-\frac{4}{8}=\frac{-3}{8}=m$

Shouldn't the slopes be the same?

9

[Pr]oblem 5 (skippable, 10 pts): Discover the precise end behavior (asymptotic behavior) of [e]ach of the following functions, as $x \to \infty$:

a. (4 pts) $f(x) = \dfrac{x^2 - 3x + 2}{x + 1}$

$$\begin{array}{r} x - 4 \\ x+1 \,\big)\, \overline{x^2 - 3x + 2} \\ -(x^2 + x) \\ \hline -4x + 2 \\ -(-4x - 4) \\ \hline 6 \end{array}$$

$\boxed{(x-4)} + \frac{6}{x+1}$

As $x \to \infty$,

the function $f(x)$ will

look like a linear function $(y = x)$

b. (4 pts) $g(x) = \dfrac{2x^2 - 3x + 2}{\sqrt[3]{x^6 + 1}}$

$$= \frac{\lim_{x\to\infty} 2x^2 - 3x + 2}{\lim_{x\to\infty} \sqrt[3]{x^6(1 + 1/x^6)}}$$

$$= \frac{\lim_{x\to\infty} 2x^2 \quad \lim_{x\to\infty} 3x + \lim_{x\to\infty} 2}{\lim_{x\to\infty} x^2 \sqrt{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \cancel{\frac{1}{x^6}}^{\,0}}}$$

$\boxed{= 2}$

As $x \to \infty$, $g(x)$ will look like the line $y = 2$ ✓

c. (2 pts) $h(x) = \sqrt{x^2 + 2x - 2}$

$$\lim_{x\to\infty} \sqrt{x^2\left(1 + \frac{2x}{x^2} - \frac{2}{x^2}\right)}$$

$$= \lim_{x\to\infty} x\sqrt{1 + \frac{2x}{x^2} - \frac{2}{x^2}}$$

$$= \lim_{x\to\infty} x \cdot \sqrt{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \cancel{\frac{2x}{x^2}}^{\,0} - \lim_{x\to\infty} \cancel{\frac{2}{x^2}}^{\,0}}$$

$$= \lim_{x\to\infty} x \cdot \sqrt{1} \quad = x$$

$-.5$ $x+1$

As $x \to \infty$, $h(x)$ will look like a linear function $(y = x)$

9.5

Problem 5 (skippable, 10 pts): Discover the precise end behavior (asymptotic behavior) of each of the following functions, as $x \to \infty$:

a. (4 pts) $f(x) = \dfrac{x^2 - 3x + 2}{x + 1}$

$$f(x) = \frac{x^2 - 3x + 2}{x+1} = (x-4) + \frac{6}{x+1}$$

$$\begin{array}{r} x - 4 \quad R\,6 \\ x+1 \,\big)\, x^2 - 3x + 2 \\ -(x^2 + x) \\ \hline -4x + 2 \\ -(-4x - 4) \\ \hline 6 \end{array}$$

as $x \to \infty$, the graph of $f(x)$ approaches $x - 4$.
the asymptote is the line $y = x - 4$.
also $x = -1$ is a vertical asymptote.

b. (4 pts) $g(x) = \dfrac{2x^2 - 3x + 2}{\sqrt[3]{x^6 + 1}}$

$$g(x) = \frac{2x^2 - 3x + 2}{\sqrt[3]{x^6 + 1}}$$

$$\cancel{g(x)} = \lim_{x \to \infty} \frac{\frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2}}{\sqrt[3]{\frac{x^6}{x^6} + \frac{1}{x^6}}}$$

$$\cancel{g(x)} = \lim_{x \to \infty} \frac{2 - \frac{3}{x} + \frac{2}{x^2}}{\sqrt[3]{1 + \frac{1}{x^6}}}$$

$$\cancel{\lim_{x\to\infty}} \frac{2 - 0 + 0}{\sqrt[3]{1 + 0}} = \frac{2}{\sqrt[3]{1}} = \frac{2}{1} = 2$$

as $x \to \infty$, the graph of $f(x)$ aproaches the line $y = 2$.

c. (2 pts) $h(x) = \sqrt{x^2 + 2x - 2}$

$$h(x) = \sqrt{x^2 + 2x - 2}$$

$$\lim_{x \to 0} \sqrt{x^2 + 2x - 2} = \cancel{\tfrac{1}{2} \lim_{x\to 0} x^2 + 2x - 2}$$

$$\lim_{x \to 0} \sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{2}{x^2}} = \lim_{x \to 0} \sqrt{1 + \frac{2}{x} - \frac{2}{x^2}}$$

$$\lim_{x \to 0} = \sqrt{1} = 1$$

~~The~~

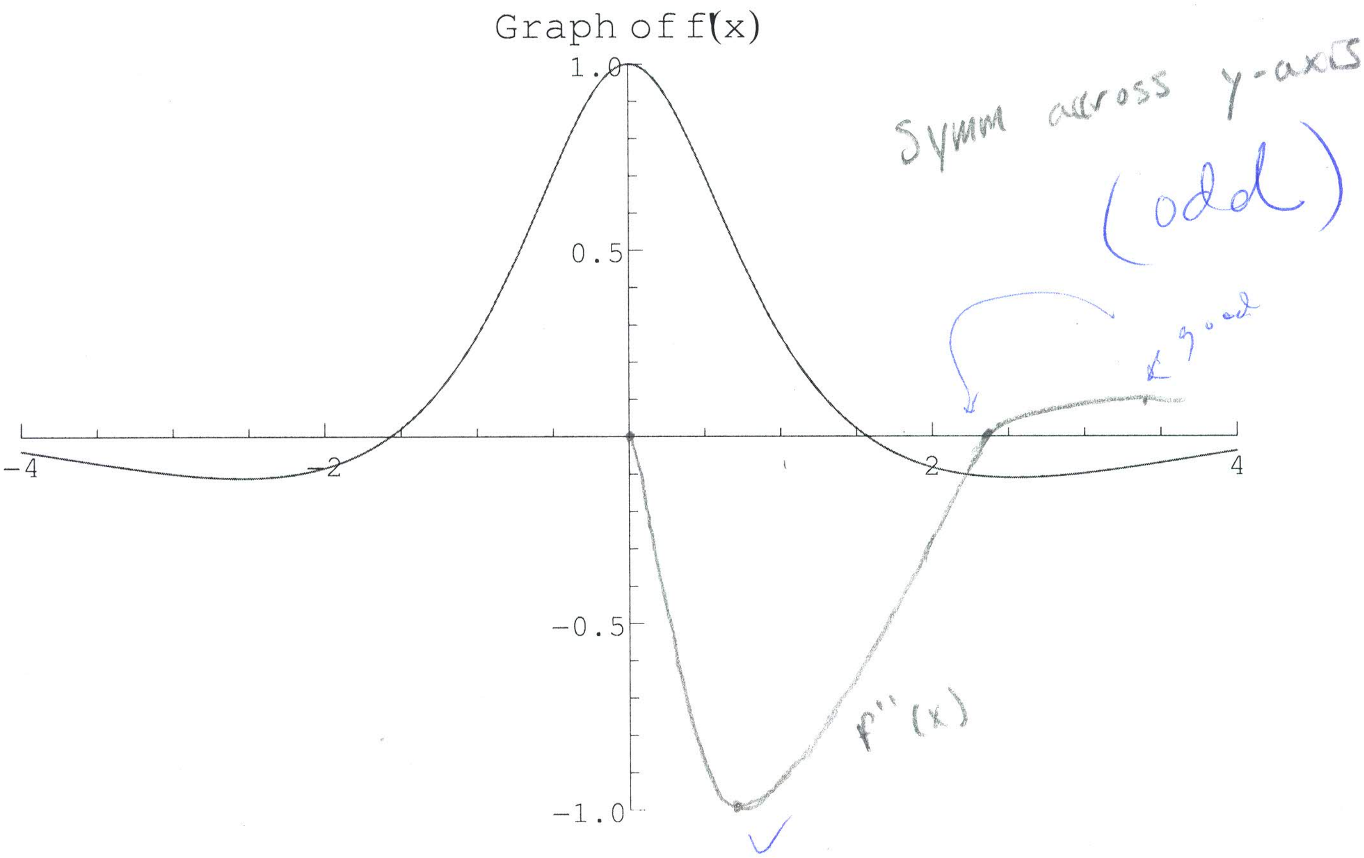
as $x \to \infty$, the graph of $f(x)$ approaches the line $y = 1$.

-1

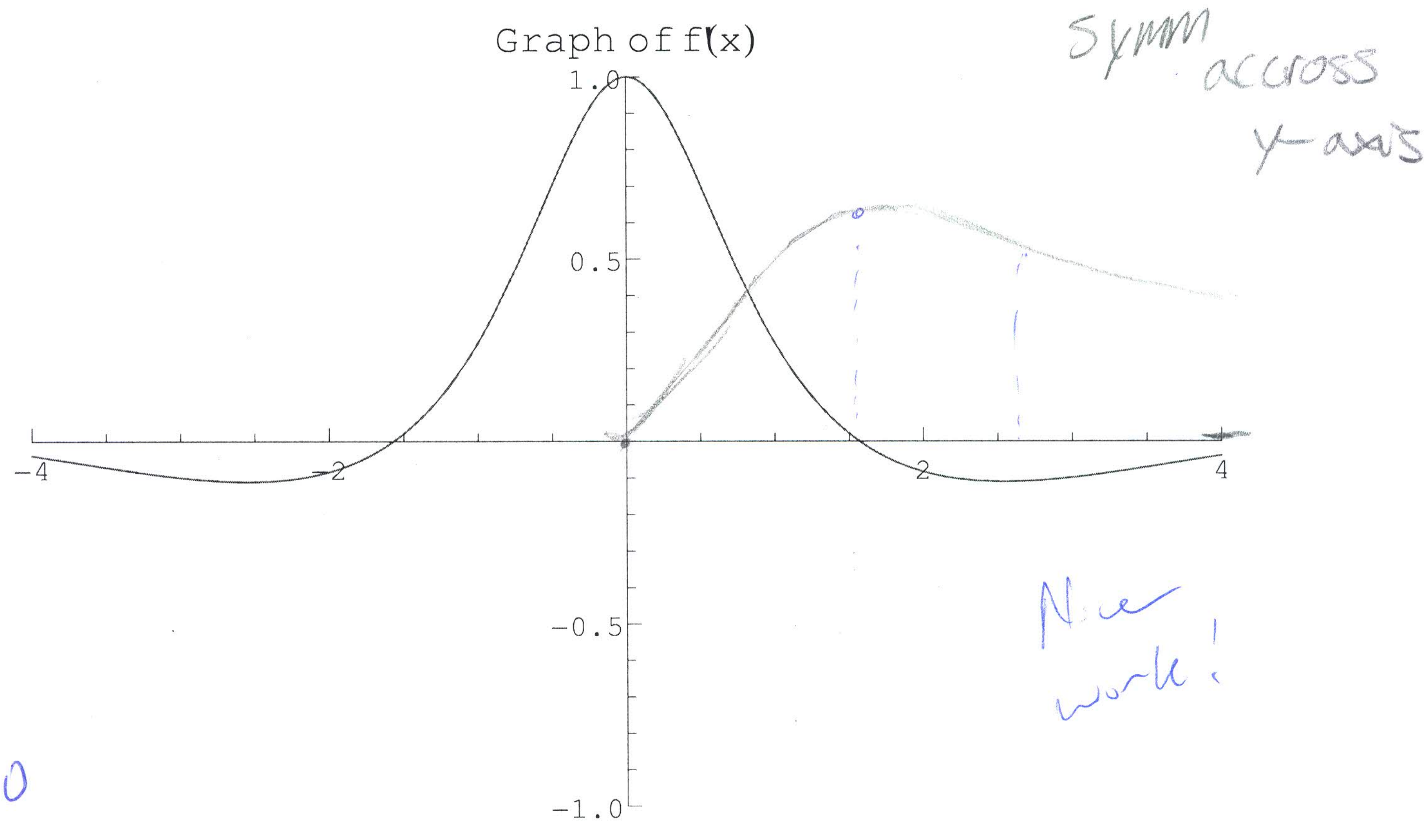
You can't just divide by x!

9

Problem 6 (skippable, 10 pts): What follows is the graph of the derivative of $f(x)$, $f'(x)$. Graph the second derivative, $f''(x)$, on the same coordinate system with $f'(x)$:



Use information you can glean from $f'(x)$ and $f''(x)$ to **carefully** sketch the graph of the function f (assume that $f(0) = 0$).



Nice work!

10

Problem 7 (10 pts): Do a study of the function $\frac{x}{x^2+1}$, culminating in a careful plot of this function on the axes below.

$f(x) = \frac{x}{x^2+1}$ $\quad f(0) = \underline{0}$ $\quad$ denom always positive $\quad$ no VA

$\lim_{x \to \pm\infty} = 0$ end behavior

$f'(x) = \frac{1(x^2+1) - (x)(2x)}{(x^2+1)^2}$

$f'(x) = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$ $\quad -x^2+1=0 \quad \sqrt{x^2}=\sqrt{1}$ $\quad$ Zeros @ $x = \pm 1$

always negative

$f'(-2) = -$

$f'(2) = -$

$f'(0.5) = +$

$f'(-1) = \frac{-1}{2}$

$f(1) = \frac{1}{2}$

$f''(x) = -2x(x^2+1)^2 - ((-x^2+1)\,2(x^2+1)(2x))$

$= -2x(x^4+2x^2+1)$

$-2x^5-4x^3-2x - ((-x^4+1)4x)$

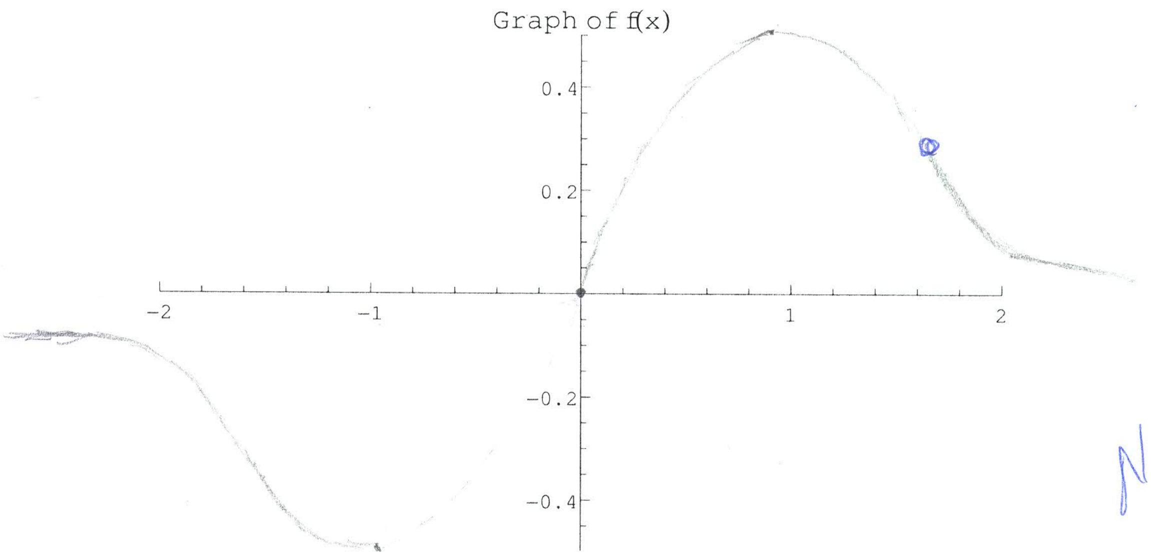
$-2x^5-4x^3-2x - (-4x^5+4x)$

$2x^5-4x^3-6x$

$2x(x^4-2x^2-3)$

$(x^2-3)(x^2+1)$ $\quad$ zeros @ $0, \pm\sqrt{3}$ $\quad f''(-1)$

$-2(1-2$



odd

Nice work

	$-\infty$		-1		0		1		$\sqrt{3}$		∞
f	0	↘	$-\frac{1}{2}$		0 ↗		$\frac{1}{2}$	↘			0
f'		—	0		+		0	—			
f''				− ∧	0				0		

10

roblem 5 (skippable, 10 pts): Discover the precise end behavior (asymptotic behavior) of each of the following functions, as $x \to \infty$:

a. (4 pts) $f(x) = \dfrac{x^2 - 3x + 2}{x + 1}$

$$\begin{array}{r} x - 4 \\ x+1 \overline{\big) x^2 - 3x + 2} \\ -\ \underline{x^2 + x} \\ -4x + 2 \\ -\ \underline{-4x - 4} \\ 6 \end{array}$$

$= \underbrace{x - 4}_{\text{end behavior as } x \to \infty} + \underbrace{\dfrac{6}{x+1}}_{\text{short behavior as gets near origin}}$

as $x \to \infty$ $f(x)$ goes to ∞ but with a slant asymptote $y = x - 4$

b. (4 pts) $g(x) = \dfrac{2x^2 - 3x + 2}{\sqrt[3]{x^6 + 1}}$ $= \sqrt[3]{x^6 \left(1 + \frac{1}{x^6}\right)} = \sqrt[3]{x^6}\,\sqrt[3]{1 + \frac{1}{x^6}}$ $\sqrt[3]{x^6}$ as $x \to \infty$ positive so $= x^2$

so $\dfrac{2x^2 - 3x + 2}{x^2 \sqrt[3]{1 + \frac{1}{x^6}}} \div x^2 = \lim_{x \to \infty} \dfrac{2 - \frac{3x}{x^2} + \frac{2}{x^2}}{\sqrt[3]{1 + \frac{1}{x^6}}}$ $\frac{1}{x^r}$ rule higher power on bottom as $x \to \infty$ $= 0$

$= \dfrac{2 - 0 + 0}{\sqrt[3]{1 + 0}} = \dfrac{2}{\sqrt[3]{1}} = 2 = y$ as the asymptote as $x \to \infty$

c. (2 pts) $h(x) = \sqrt{x^2 + 2x - 2}$ ← complete the square for the slant asymptote

$= \lim_{x \to \infty} \sqrt{x^2 \left(1 + \frac{2x}{x^2} - \frac{2}{x^2}\right)}$ $= \lim_{x \to \infty} \dfrac{\sqrt{1 + \frac{2}{x^2} - \frac{2}{x^2}}}{\frac{1}{x}}$ $\frac{1}{x^r}$ $x \to \infty$ $= 0$

$= \lim_{x \to \infty} \sqrt{x^2}\,\sqrt{1 + \frac{2}{x^2} - \frac{2}{x^2}}$ $\dfrac{\sqrt{1}}{0}$

-.5

$\lim_{x \to \infty} = 0$

$x \to \infty$ positives so =

should = 1

$= \lim_{x \to \infty} \dfrac{x \sqrt{1 + \frac{2}{x^2} - \frac{2}{x^2}}}{1} \div x$

9.5

Problem 4 (skippable, 10 pts): Consider the unit circle ($x^2+y^2=1$) at the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

a. (6 pts) Demonstrate implicit differentiation to find the equation of the tangent line at that point. Sketch in your tangent line.

$$\frac{d}{dx}\left(x^2+y(x)^2=1\right)$$

$$2x + 2y(x)\cdot y'(x) = 0$$

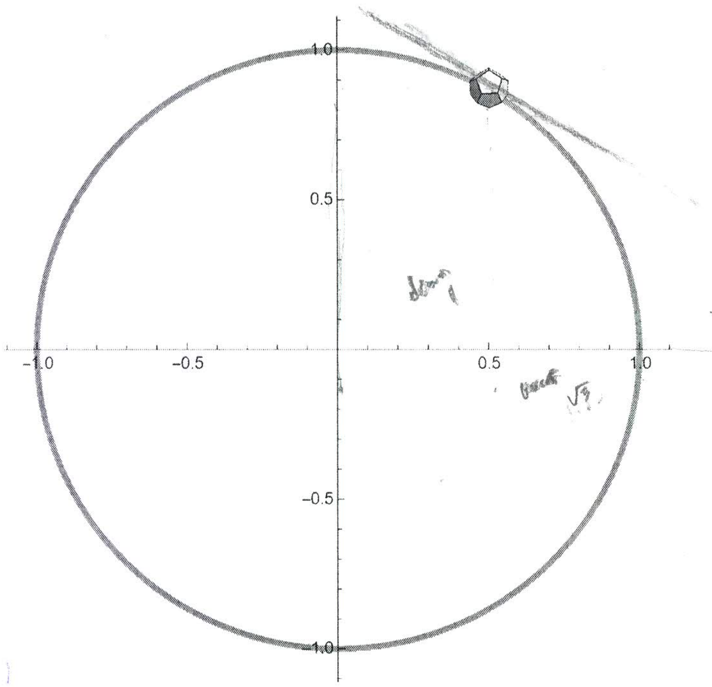
$$2y(x)\cdot y'(x) = -2x$$

$$y'(x) = \frac{-2x}{2y} \qquad y'(x) = \frac{-x}{y}$$

$$y'(x)\Big|_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

is the slope

$\approx -.577$



Tangent line:

$$y - \frac{\sqrt{3}}{2} = -\frac{1}{\sqrt{3}}\left(x-\frac{1}{2}\right)$$

$$y = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\left(x-\frac{1}{2}\right)$$

(4 pts) Now do the same problem **explicitly**:

a. Solve the equation of the unit circle for y as the appropriate **explicit** function of x.

$x^2+y^2=1 \qquad \sqrt{y^2} = \sqrt{1-x^2} \qquad y = \pm\sqrt{1-x^2}$ using top half so $\rightarrow$ (concave down)

$$y = \sqrt{1-x^2}$$

$$y' = -\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(2x) = \frac{-2x}{2(1-x^2)^{\frac{1}{2}}} = \frac{-x}{(1-x^2)^{\frac{1}{2}}}$$

b. Differentiate your expression $y(x)$ to obtain the slope of the tangent line at $x=\frac{1}{2}$.

$$y'\left(\tfrac{1}{2}\right) = \frac{-\frac{1}{2}}{\left(1-\left(\frac{1}{2}\right)^2\right)^{\frac{1}{2}}} = \frac{-\frac{1}{2}}{\sqrt{0.75}} = -\frac{1}{2}\cdot\frac{1}{\sqrt{0.75}} \approx -0.5774 \quad \text{same}$$

Good!

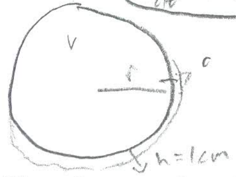
Problem 3(10 pts): Related Rates: A ferris wheel spinning at 90 mph flies off its stand and hits an ice cream truck traveling 30^o north-by-northwest at 12 ft/s, as an extension ladder slides off the truck into the southbound lane at half that speed. The driver of the truck (a mathematician) dies while considering the following related rates problem:

An oil slick is spreading out in a circle from an oil drilling platform, which is leaking oil into a calm sea. The slick is one cm thick. The oil is spilling at the rate of 1000 liters[1] per second.

a. (2 pts) What is the volume of the slick (a cylinder of circular cross section of radius r and height h)? (Draw it. I'll sell the answer to you, if you can't figure it out.)

$\frac{dV}{dt} = 1000$ L/sec

area $= \pi r^2$
vol $= \pi r^2 \cdot h$



$V = \pi r^2 \cdot h$ h= 1cm

$V = \pi r^2 \cdot 1cm$

b. (4 pts) How fast is the slick's radius increasing when the slick is 25 meters in radius?

want $\frac{dr}{dt}$ change in r

when r = 25 m in radius

$r = 25m \times \frac{100cm}{1m} = 2500$ cm

$V = \pi (r)^2 \cdot 1cm$

$\frac{dV}{dt} = 2\pi r(t) \cdot \frac{dr}{dt}$

$1000\ L/s = 2\pi r \cdot \frac{dr}{dt}$

height

$\frac{1000\ L}{s} \cdot \frac{1000\ cm^3}{L} = 1000{,}000\ cm^3/s = 2\pi(2500\ cm)\ 1cm \cdot \frac{dr}{dt}$

$1000000\ cm^3/s = 5000\pi\ cm^2\ \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{200}{\pi}\ cm/s$

the radius is increasing at a rate of 200π cm per second when the radius is 25 m

-.5

c. (4 pts) How fast is the slick's area changing at the very same moment?

wait

$A = \pi r^2 \cdot h$ h = 1cm

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$A = \pi r^2 + h \cdot 2\pi r$

$\frac{dA}{dt} = 2\pi r\ \frac{dr}{dt} \cdot 2\pi \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi \cdot 2500cm \cdot 200\pi\ cm/s$

GYE

$\frac{dA}{dt} = 1{,}000{,}000\pi\ cm^2/s$

area increasing $1000000\pi\ cm^2/s$ when radius is 25 m

9.5

[1] A liter is 1000 cm^3.

Problem 2(10 pts): Assume that f and g are differentiable at x, and that $p(x) = f(x) + g(x)$.

a. (7 pts) Use the limit definition of the derivative to demonstrate the sum rule – that is, that

$$p'(x) = f'(x) + g'(x)$$

$P(x) = f(x) + g(x)$

$$p'(x) = \lim_{h \to 0} \frac{(f(x+h) + g(x+h)) - (f(x) + g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) \quad \text{sum rule}$$

$$= \underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{f'(x)} + \underbrace{\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}}_{g'(x)}$$

$p'(x)$

So $p'(x) = f'(x) + g'(x)$

b. (3 pts) Suppose that f and g above are both differentiable on $[a, b]$, where a and b are real numbers. What does the Extreme Value Theorem guarantee about the function p?

diff + cont $[a,b]$

if f and g are differentiable on $[a,b]$ that means both f and g are continuous functions. That also means p is continuous, since adding two continuous fns. By the Extreme Value Theorem p must have an absolute max and absolute min on $[a,b]$, because it is a closed interval + continuous even if the max and min are the same number, they are still absolute

$[-1, 1]$

Great!

10

Problem 1 (skippable, 10 pts): Demonstrate your differentiation prowess (simplify where possible):

a. Quotient rule: $f(x) = \dfrac{\cos(x)}{x^2 - 3}$ $f'(x) = \dfrac{-\sin(x)(x^2-3) - (\cos(x)\cdot 2x)}{(x^2-3)^2} = \dfrac{-\sin(x)(x^2-3) - 2x\cos(x)}{(x^2-3)^2}$ ✓

b. Chain rule: $g(x) = \cos(x^2 - 3)$ $g'(x) = -\sin(x^2-3)(2x)$ ✓

c. Chain rule: $p(x) = (\cos(x))^2 - 3$ $p'(x) = 2(\cos(x))\cdot(-\sin(x))\cdot 1$ ✓

d. Product rule: $q(x) = \cos(x)(x^2 - 3)$ $q'(x) = -\sin(x)(x^2-3) + \cos(x)(2x)$ ✓