Section Summary: 2.4

a. **Definitions** Understanding derivatives of trig functions begins with $\sin(x)$ and $\cos(x)$. You should first remind yourself of how these are defined (see Appendix D), and how they relate to the circle.

You should know the definitions of the various "secondary" trig functions, which are defined in terms of sin and cos:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

b. **Theorems** The most important limit we will need is this one:

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

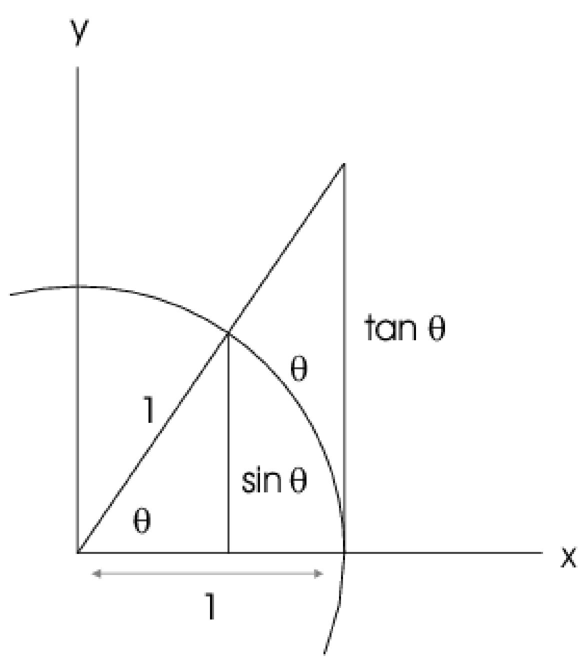

Figure 1: The circle and the circular functions

This makes sense graphically, since if we zoom in on the function $\sin x$ as $x \to 0$, it appears more and more like the line $y = x$. Thus the ratio should be approaching 1, since they're getting more and more similar as $x \to 0$.

We can demonstrate that this limit is correct using the squeeze theorem, one important identity, and a little trigonometry (explained in Appendix D). In particular, the length of a sector of a circle (a little piece of arc) is $a = r\theta$, where r is the radius of the circle, and θ is the angle subtending the arc of length a (see Figure **??**).

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

c. Properties/Tricks/Hints/Etc.

Identity:

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin(x + h) = \sin x \cos h + \cos x \sin h$$

This identity comes from the Appendix D, which you may want to consult if you're a little shaky on your trig functions.

The author notes that the derivatives of the "co" functions are the ones with the negative signs.

d. Summary

In this case we use a few tricks and identities to find the derivative functions for the trigonometric functions. Since all these functions are defined in terms of sines and cosines, if we know the derivatives for these the others can be derived using the quotient rule.