Section Summary: 3.3

a. Definitions

- **concave upward** on interval I: the graph of f lies above all of its tangents on the interval I. ("Bowl-like")
- **concave downward** on interval I: the graph of f lies below all of its tangents on the interval I. ("Umbrella-like")
- **inflection point**: Point P on a curve is an inflection point if the curve changes concavity at P (goes from a bowl to an umbrella, or *vice-versa*).

  "There is a point of inflection at any point where the second derivative changes sign." (p. 218)

b. Theorems

- **Increasing/Decreasing Test**:

  If $f'(x) > 0$ on $[a, b]$, then f increases on $[a, b]$;

  if $f'(x) < 0$ on $[a, b]$, then f decreases on $[a, b]$.

- **The first derivative test**: Let c be a critical number of continuous function f.

  If f' changes sign from positive to negative at c, then f has a local maximum at c.

  If f' changes sign from negative to positive at c, then f has a local minimum at c.

  If f' does not change sign at c, then f has neither a max nor a min at c.

- **Concavity test**:
  If $f'' > 0$ for all x in interval I, then f is concave up on I;
  if $f'' < 0$ for all x in interval I, then f is concave down on I.

  Just remember the two types of parabolas: bowls and umbrellas.
  - Bowl: $y = x^2$, so $y''(0) = 2 > 0$, and the curve is concave up;
  - umbrella: $y = -x^2$, so $y''(0) = -2 < 0$, and the curve is concave down.

- **Second derivative test**: Suppose f'' is continuous near c.
  If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c;
  If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

  The second derivative test is inconclusive if $f''(c) = 0$. Examples to illustrate this: x^3 (inflection at $x = 0$); x^4 (minimum at $x = 0$).

c. Properties/Tricks/Hints/Etc.

You'll notice that there are some nice tables which are created to show the sign of the derivative and hence indicate the direction (increasing/decreasing) of the function's graph; this is a graphing aid.

d. Summary

Knowledge of f' and f'' inform us about critical aspects of f (increasing/decreasing, extrema, points of inflection, concavity).