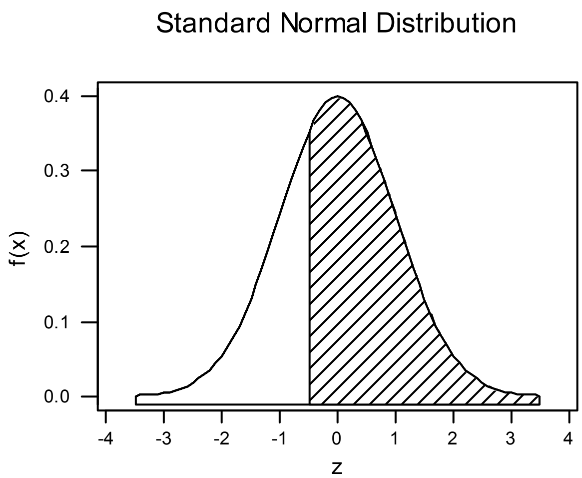
Normal Distribution Homework Solutions (pages 245-246)

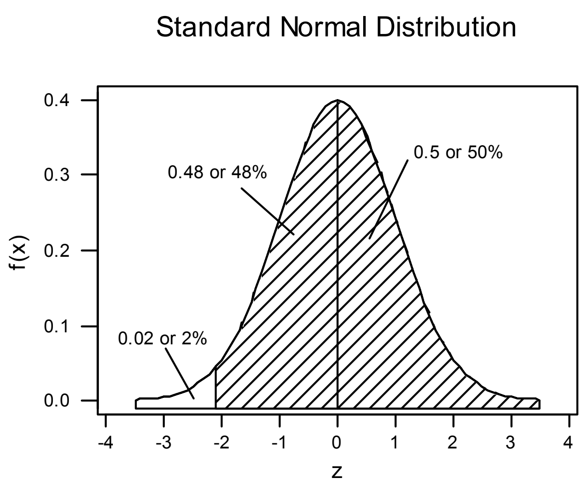
8.40 X is the lifetime for a randomly selected bulb where X ~ Normal($\mu$ = 5100, $\sigma$ = 200).

$$P(X > 5000) = P\left(\frac{X-5100}{200} > \frac{5000-5100}{200}\right) = P(Z > -0.5)$$

$$= P(-0.5 < Z < 0) + P(Z > 0) = 0.1915 + 0.5 = \mathbf{0.6915}$$




8.41 Working backwards, we want to find x such that P(X > x) = 0.9800.




So in the standard normal picture, what value of z gives P(Z > z) = 0.98? Look for 0.4800 in the body of Table 3. Use the closest value which is 0.4798. This corresponds to the little z value of 2.05, but recall in our picture, we are below 0 (negative numbers), so we add a negative sign.
Thus, P(Z > -2.05) = 0.98. Notice we are looking for $z_{0.98} = -z_{0.02}$.

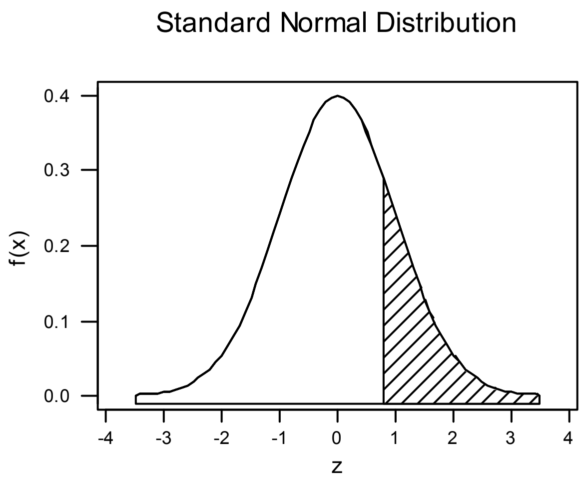
Now, we "untransform" our -2.05 to find the value of x we are seeking.

Since $z = \frac{x-5100}{200}$ we can put in -2.05 for z and solve for x.

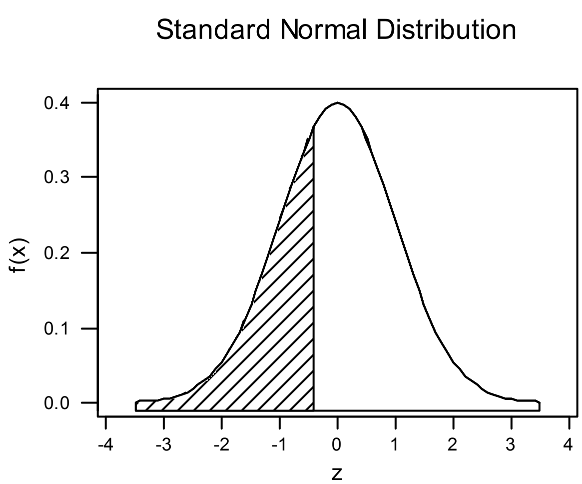
$-2.05 = \frac{x-5100}{200}$ which implies that $200(-2.05) + 5100 = x$ and hence x = 4690.

We would advertise that the bulbs last **4,690 hours**.

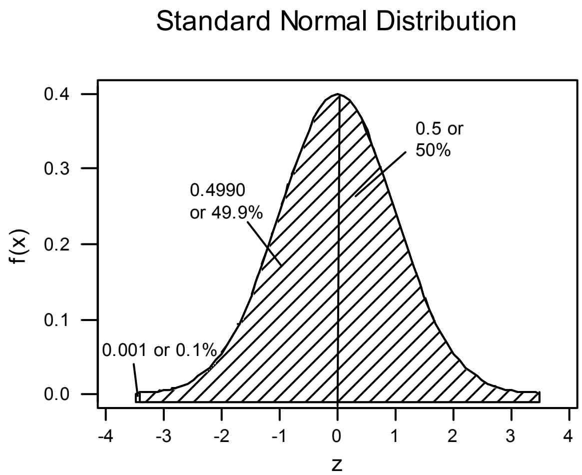
8.42 a P(X > 12000) = $P\left(\frac{X-\mu}{\sigma} > \frac{12000-10000}{2400}\right)$ = P(Z > .83) = .5 - P(0 < Z < .83) = .5 - .2967 = .2033




b P(X < 9000) = $P\left(\frac{X-\mu}{\sigma} < \frac{9000-10000}{2400}\right)$ = P(Z < -.42) = .5 - P(0 < Z < .42) = .5 - .1628 = .3372



8.43 $P(0 < Z < z_{.001}) = .5 - .001 = .4990$; $z_{.001} = 3.08$. Then, $z_{.001} = \frac{x-\mu}{\sigma}$; $3.08 = \frac{x-10000}{2,400}$; x = **17,392.**




8.44 a $P(X > 70) = P\left(\frac{X-\mu}{\sigma} > \frac{70-65}{4}\right) = P(Z > 1.25) = .5 - P(0 < Z < 1.25)$

$= .5 - .3944 = .1056$

b $P(X < 60) = P\left(\frac{X-\mu}{\sigma} < \frac{60-65}{4}\right) = P(Z < -1.25) = .5 - P(0 < Z < 1.25)$

$= .5 - .3944 = .1056$

c $P(55 < X < 70) = P\left(\frac{55-65}{4} < \frac{X-\mu}{\sigma} < \frac{70-65}{4}\right) = P(-2.50 < Z < 1.25)$

$= P(0 < Z < 2.50) + P(0 < Z < 1.25) = .4938 + .3944 = .8882$

8.45 a $P(X < 70000) = P\left(\frac{X-\mu}{\sigma} < \frac{70000-82000}{6400}\right) = P(Z < -1.88) = .5 - P(0 < Z < 1.88)$

$= .5 - .4699 = .0301$

b $P(X > 100000) = P\left(\frac{X-\mu}{\sigma} > \frac{100000-82000}{6400}\right) = P(Z > 2.81) = .5 - P(0 < Z < 2.81)$

$= .5 - .4975 = .0025$

8.48 $P(X > 8) = P\left(\frac{X-\mu}{\sigma} > \frac{8-7.2}{.667}\right) = P\ Z > 1.2) = .5 – P(0 < Z < 1.2) = .5 - .3849 = .1151$

8.49 $P(0 < Z < z_{.25}) = .5 - .25 = .2500$; $z_{.25} = .67$;

$z_{.25} = \frac{x-\mu}{\sigma}$; $.67 = \frac{x-7.2}{.67}$; x = 7.65 hours

8.54 a $P(X > 30) = P\left(\frac{X-\mu}{\sigma} > \frac{30-27}{7}\right) = P(Z > .43) = .5 - P(0 < Z < .43)$

$= .5 - .1664 = .3336$

b $P(X > 40) = P\left(\frac{X-\mu}{\sigma} > \frac{40-27}{7}\right) = P(Z > 1.86) = .5 - P(0 < Z < 1.86)$

$= .5 - .4686 = .0314$

c $P(X < 15) = P\left(\frac{X-\mu}{\sigma} < \frac{15-27}{7}\right) = P(Z < -1.71) = .5 - P(0 < Z < 1.71)$

$= .5 - .4564 = .0436$

d $P(0 < Z < z_{.20}) = .5 - .20 = .3000$; $z_{.20} = .84$

$z_{.20} = \frac{x-\mu}{\sigma}$; $.84 = \frac{x-27}{7}$; x = 32.88

8.56 a $P(X < 10) = P\left(\frac{X-\mu}{\sigma} < \frac{10-16.40}{2.75}\right) = P(Z < -2.33) = .5 - P(0 < Z < 2.33)$

$= .5 - .4901 = .0099$

b $P(-z_{.10} < Z < 0) = .5 - .10 = .4000$; $-z_{.10} = -1.28$

$-z_{.10} = \frac{x-\mu}{\sigma}$; $-1.28 = \frac{x-16.40}{2.75}$; x = 12.88