MAT212 Test 2 (Spring 2004): sections 9.1-9.3, 10.1-10.4, 11.1-3

Name:

INSTRUCTIONS

1. ALL ANSWERS to be graded MUST BE ON THIS TEST.
2. Show ALL WORK to receive ANY CREDIT.
3. A normal table is attached to your test.
4. Points for each problem are in parentheses.

Problem 1 (15 pts) The issue: some people at work think that they should strike over their compensation. To gauge interest in a strike, they take a sample of 100 of their co-workers. Assume that, in actuality, 60% of workers support a strike.

1. (5 pts) What is the probability that 66 (or more) in the sample of 100 workers would favor a strike?

2. (5 pts) What is the probability that 50 (or fewer) in the sample of 100 workers would favor a strike?

3. (5 pts) What conditions should be met in order for the calculations above to be valid?

Problem 2 (20 pts) In a manufacturing process, an assembly lines produces a product with standard deviation $\sigma = 5$. Consider the following representative times (in minutes):

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22.6 38.8 23.4 28.5 17 18.9 16.1 10.6 21.3 32 33.7 28.7 27.2 34.9 29.4
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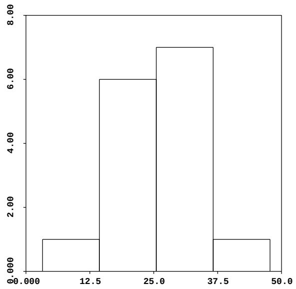

1. (10 pts) Use the data to provide your best estimate of the mean time of manufacturing, with a significance level of .05. Comment on the validity of the procedure.

2. (5 pts) If you wanted to estimate the mean to within one minute, with 95% confidence, what minimum sample size would be required?

3. (5 pts) Suppose that you suspect that the true mean is less than 25. Test your hypothesis using this data, with 90% confidence.

Problem 3 (15 pts) A random variable X has a mean of μ and a standard deviation of σ. Its distribution is anything but normal: it is bi-modal, dramatically skewed right, etc.

We create a new random variable $\overline{X}$ by sampling from the X distribution (using a random sample of 200 values).

1. (5 pts) What is the mean of the sampling distribution of $\overline{X}$?

2. (5 pts) What is the standard deviation of the sampling distribution of $\overline{X}$?

3. (5 pts) Describe the distribution of $\overline{X}$ (by comparison with X).

Problem 4 (15 pts) Your teenage son has been acting strangely, and you suspect that he may be experimenting with drugs. You are considering making a test of your suspicions.

1. (5 pts) Describe legitimate null and alternative hypotheses that might apply in this situation.

2. (5 pts) Describe type I and type II errors given your hypotheses. What are the consequences of making these errors?

3. (5 pts) Describe the relative values you would want α and β to have considering the consequences of the errors.

Problem 5 (20 pts) As part of a marketing survey, someone in your group conjectures that the mean monthly gasoline bill for a certain population is more than \$110/month. A random sample of 85 gasoline bills shows a mean of \$115. Suppose we know that the standard deviation of the population is \$25.

1. (15 pts) Set up and carry out an appropriate test of hypothesis, with a confidence level of 95%. Use both the rejection region approach, and the p-value approach.

2. (5 pts) Using a picture, illustrate the difference between the rejection region approach and a p-value approach to hypothesis testing. Why might one be preferable to the other?

Problem 6 (15 pts) Three short stories!

1. (5 pts) In a right-tailed test of hypothesis, you obtain a standard normal $Z = 8$. Are you likely to require a table to arrive at your conclusion?

2. (5 pts) Is the sample mean an unbiased estimator of the population mean μ? Why or why not?

3. (5 pts) Suppose our null hypothesis is $\mu = 0$, our alternative is $\mu < 0$, and we calculate a sample statistic of $\overline{x} = 3$. What do we conclude?

Formulas for Test 2

$$\overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$n = \left(\frac{z_{\alpha/2}\sigma}{W} \right)^2$$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{n}}$$

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$